White Paper

Risk Optimised Investing in Equity Markets

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Executive Summary

Recent academic research shows that the risk-return relationship in most equity markets is too flat: low risk investment strategies yield comparable – and often superior – risk adjusted returns to investing in capitalization weighted benchmarks or in high beta stocks.

Explanations for this "anomaly" include a focus on tracking error and constraints on leverage for institutional investors. Furthermore, a series of behavioural biases drive market prices away from their fundamental value cause bubbles and crashes, and creating excess volatility that has no return compensation. Finally, investor disagreement about the fundamental stock price makes investors price in the option to resell the asset to future optimists.

Investors in less risky stocks enjoy the so called compounding effect: the compounded return of a low volatility strategy will be higher than the compounded return of a high volatility strategy with the same arithmetic return. This is because a loss of percentage is not offset by a gain of a similar percentage.

Whilst many professionals can identify the strengths of such an approach, few know how to design an optimal low risk equity portfolio.

Simple minimum variance strategies using the sample covariance matrix have poor out of sample performance because of the estimation error in the sample. A robust, low risk equity portfolio however, takes into account the three key stylized facts of financial returns: volatility clustering, fat tails and nonlinear dependence.

Magnitudes of volatility tend to cluster together. This means that generally one will find a series of days of high volatility followed by a series of days of low volatility. Large negative returns occur much more frequently than would be expected based on a normal distribution, explaining the need for inclusion of fat tail estimates. Return dependence is time-varying as a function of market regimes and consequently asymmetric.

The R-Equity engine of Finvex Group offers an advanced approach to low risk equity portfolio optimisation.

The starting point is a dynamic screening of the universe based on a series of individual risk characteristics, measuring the time-varying downside and systematic risks of the assets. These estimation methods are incredibly robust and elements from operational research are used to represent standard portfolio constraints.

Accurate estimates on the conditional co-moment matrices are achieved through a variety of GARCH models (Bollerslev, 2008). For the correlation, the co-skewness and the co-kurtosis matrices, R-Equities uses so called "structured" estimators. Time-varying dependence is incorporated through the use of dynamic conditional correlation models.

The conditional co-moment estimates are integrated in a portfolio optimiser that can handle multiple non-linear objectives such as minimum risk, minimum tracking error, minimum turnover, maximum skewness, highest risk diversification, together with flexible constraints on sector-country weights, maximum allowed risk, tracking error and turnover.

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1. Introduction

One of the accepted principles of financial theory is that stocks with a higher risk should have a higher expected return. Recent academic research brings this assumption into question as it shows that historically, the risk-return relationship is too flat: low risk investment strategies yield comparable – and often superior – risk adjusted returns to investing in capitalization weighted benchmarks or in high beta stocks.

To this end, Rabobank International has commissioned Finvex Group to author this white paper, which discusses current academic literature and theory and describes the components of a successful risk optimised investment strategy.

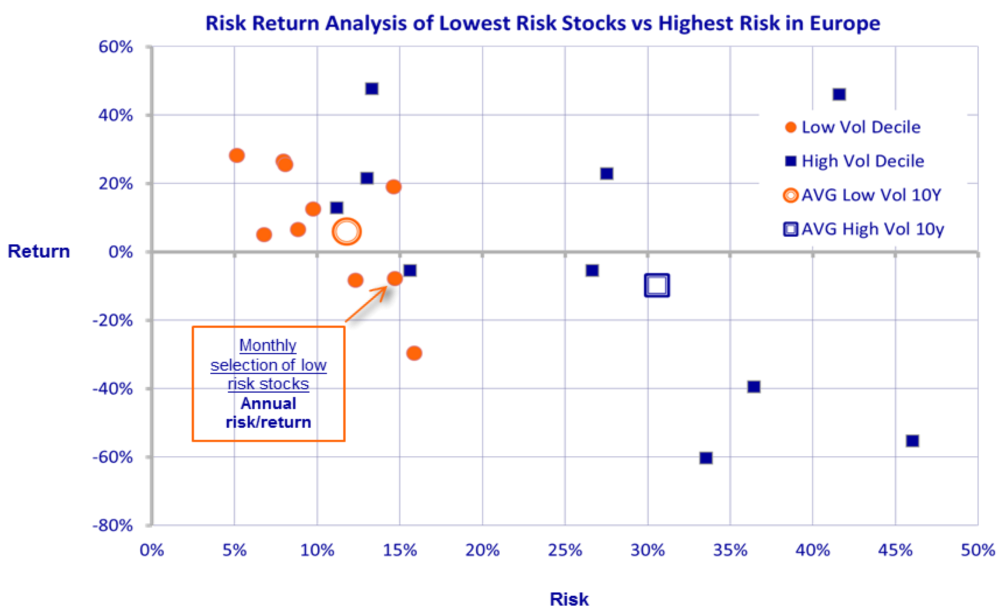
The Finvex Group provides specific financial services to investment managers, financial advisors, banks, insurers and research houses. Finvex focuses on designing robust financial investment strategies in all asset classes with the focus to reduce investment risks. They combine academic research with advanced and proprietary technology to analyse all types of financial risks, with the objective of adding stability to overall investment portfolios.

2. On the return-free risk of the market portfolio

We define the "return-free risk of a portfolio" as the difference in risk between that portfolio and the corresponding mean-risk efficient portfolio. Several recent academic papers have illustrated return-free risk taking of a market portfolio. Clarke, de Silva and Thorley (2006) found that minimum variance portfolios based on the 1000 largest U.S. stocks over the 1968-2005 period achieve a volatility reduction of about 25%, while delivering comparable or even higher average returns than the market portfolio.

Based on a simple historical risk measure – namely volatility – we divide the universe of European large caps in deciles over the period 2000-2010 on a monthly basis. This plots the annual risk and return of the low and high volatility deciles (orange dots and blue squares respectively).

The chart shows that the empirical relationship between return and risk is too flat and runs contrary to the theoretical relationship in that higher risk was not rewarded by higher return.



Source: Finvex Group

Repeating this exercise for the last five decades using US data yielded similar results for each decade, except for the 90's where the absolute return of high risk stocks was better although not per unit of risk.

There are several explanations why this result runs contrary to the assumed principle that higher risk should equate to a greater return.

First, institutional investors focus on tracking error which discourages investments based on total market risk measures (Baker, Bradley and Wurgler, 2011). Investing in low-risk stocks actually increases the tracking error so institutional investors often consider low-risk stocks as unattractive.

Second, many investors are constrained in the leverage they can take and therefore over-weight risky securities instead of using leverage, leading to lower risk-adjusted returns for risky high-beta assets (Frazzini and Pedersen, 2010).

Third, the behavioural biases of investors drive market prices away from their fundamental value. One behavioural bias is the lottery ticket effect: risk-seeking investors buy high-risk stocks hoping to reap high returns quickly. For example, asset managers have an incentive to buy more risky stocks as they hope to outperform the market. Another is an attention bias: investors have a preference for buying stocks that are in vogue (i.e. receiving media coverage).

A further bias is overconfidence: most investors believe that they take better decisions than the average investor and as a consequence trade too excessively. A final bias is representativeness: investors assume that stocks that performed well in the past will continue to perform well in the future. All these biases cause bubbles and crashes which create excess volatility that has no return compensation (Shiller, 2000).

Finally, when there is investor disagreement about the stock fundamental price, investors' price in the option to resell the asset to future optimists. The higher the volatility of the stock, the higher the overpricing tends to be (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003; Hong and Sraer, 2011).

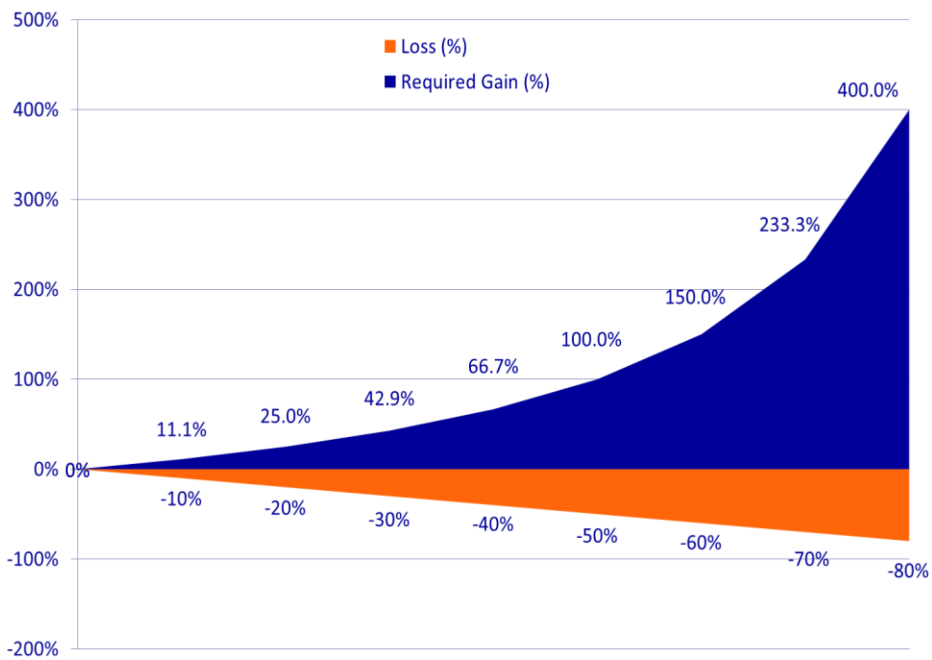
These explanations are so deeply rooted in today's financial markets that the return-free risk anomaly of the market portfolio can be expected to last for many years to come.

3. Stability Compounds

Since the seminal work of Harry Markowitz (1952 onwards) the gains of portfolio diversification are well known. A less well known advantage of focussing on risk rather than return is that the compounded return of a low volatility strategy will be higher than the return of a high volatility strategy with the same arithmetic average return.

This follows from the asymmetric impact of positive and negative returns. If a portfolio loses 20% in one month, and gains 20% the next month, the portfolio will still have an overall loss of 4%. To recover fully from the loss of 20%, the portfolio actually needs to gain 25% the next month rather than just 20%.

The asymmetric impact of returns increases with volatility. Assume that a portfolio loses 50% in one month: to recover this loss the portfolio needs to gain 100% the next month. In the graph below it can be seen how the gains required to recover from a loss increase exponentially with the size of the loss.



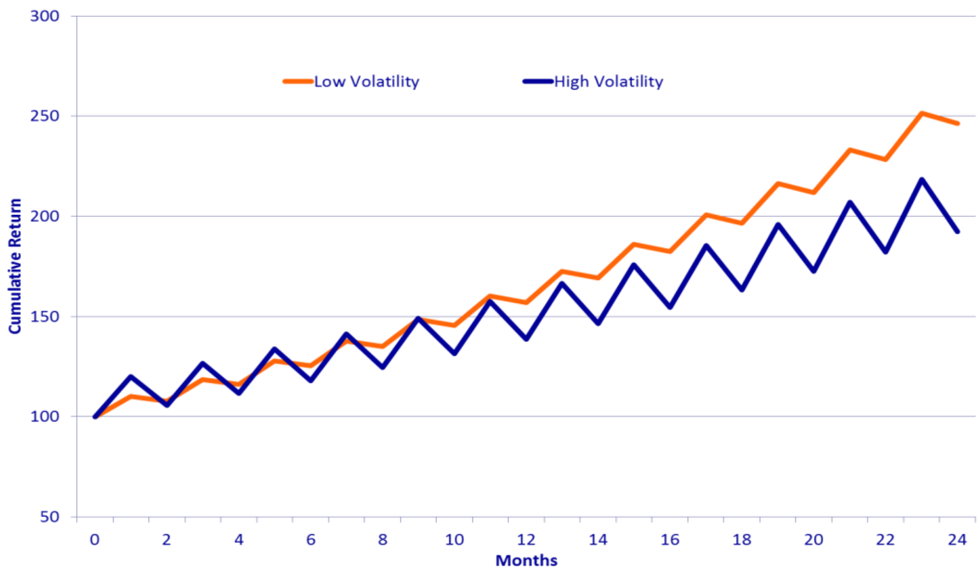
Source: Finvex Group

As a consequence, the compounded return of low volatility strategy will be higher than the compounded return of a high volatility strategy with the same arithmetic return. This can be illustrated with a simple example.

Assume 2 portfolios: one portfolio has alternating monthly returns of 10% and -2%, the second has alternating monthly returns of 20% and -12%. The average return of both portfolios is the same (4% per month) while the volatility of the second portfolio is higher but the performance over the longer term of first portfolio with lower volatility is better.

The graph below shows that the compounded return of the low volatility strategy is higher than for the high volatility strategy: the annualised compounded return of the low volatility strategy is 57% compared to 39% for the high volatility strategy.

One can also see that over the first months the high volatility strategy outperforms the low volatility strategy, but the latter catches up and outperforms over longer horizons.



Source: Finvex Group

Lowering volatility clearly has a positive impact on returns over longer periods:

- **Between +10% and -2% every month = 4% average**
- **Between +20% and -12% every month = 4% average**

4. Three stylized facts of financial returns

How can investors avoid return-free risk? Several key articles have shown that - because of the flatness of the risk-return relationship - a forward looking minimum risk strategy has significantly less risk than a value-weighted benchmark whilst offering a comparable return (see e.g. Clarke, de Silva and Thorley, 2006, Behr, Güttler and Miebs, 2008).

The important caveat is that this requires an appropriate implementation of the strategy.

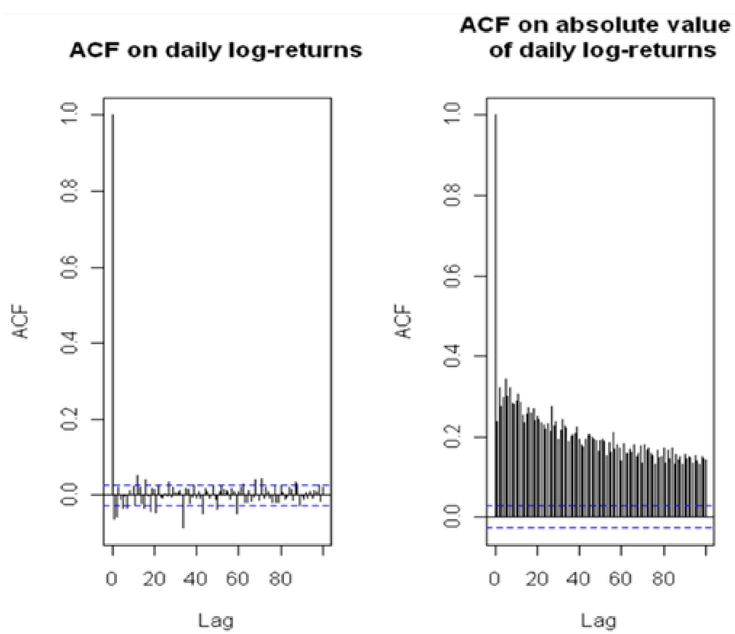
A simple minimum variance strategy using the sample covariance matrix results in poor out of sample performance because of the estimation error in the sample covariance (Jagannathan and Ma, 2003; Ledoit and Wolf, 2003).

It is also important to take into account the presence of volatility clustering, fat tails and non-linear dependence, collectively known as the three key stylized facts of financial returns (Danielsson 2011).

4.1 Volatility Clustering

Volatility clustering describes a phenomenon in which magnitudes of volatility tend to cluster together. This means that one generally will find a series of days of high volatility followed by a series of days of low volatility.

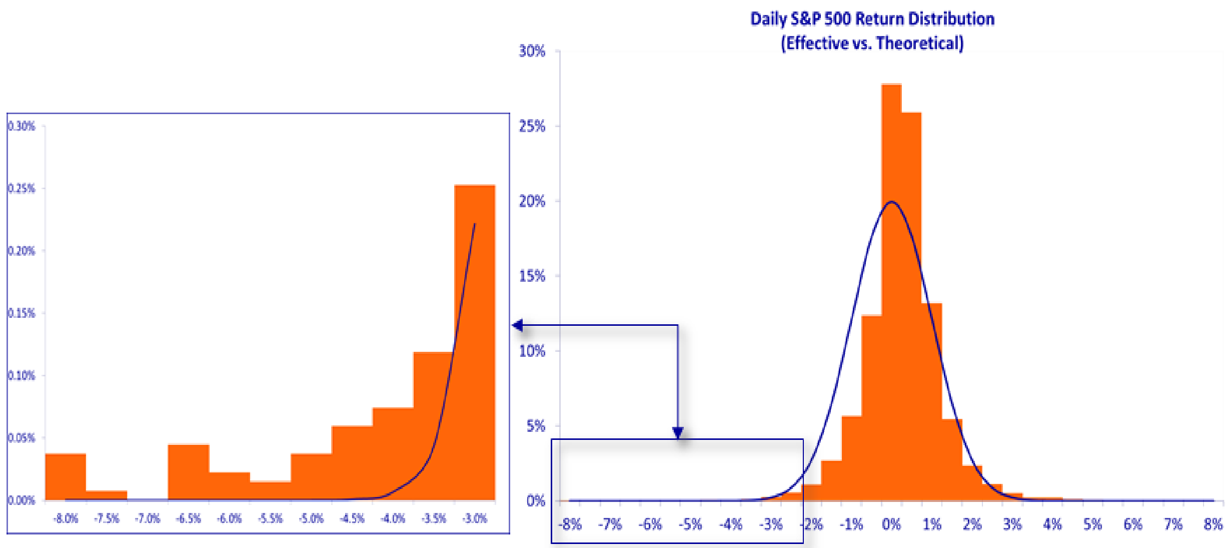
This can be seen by looking at the autocorrelation function of the S&P 500: daily log-returns (left graph) show no correlation, whereas the absolute value of daily log returns (right graph) shows substantial auto-correlation. The dashed lines in the ACF graphs are the 95% Bartlett confidence bands: autocorrelations that exceed these bands are statistically significant.



Source: Finvex Group, June & August 2011

4.2 Fat Tails

The second key fact is the presence of fat tails. This can be illustrated with the graph below which shows the return distribution of the S&P 500 since 1960.



Source: Finvex Group

When focusing on the left tail (cfr. graph left), it can be seen that large negative returns occur much more frequently than would be expected based on a normal distribution.

4.3 Non-linear dependence

The final fact to consider is the presence of nonlinear dependence: this addresses how multivariate returns relate to each other. This dependence is time-varying in function of market regimes and asymmetric: dependence tends to be higher between extreme negative returns than between extreme positive returns.

When estimating risk, it is crucial to take into account these three stylized facts of financial returns. How this is achieved in practice is described in the following paragraph.

5. Adequate risk forecasting

Important gains in risk reduction - without sacrifices in return - can already be achieved by simply restricting the investment universe to the stocks with lowest risk, as proxied by a joint measure of systematic risk and the stock specific downside risk. For many time periods, the portfolios constructed using these simple sorts have outperformed the market portfolio (Baker, Bradley and Wurgler 2011).

In this reduced investment universe, further gains are realized by optimizing a forward looking estimate of portfolio risk. Portfolio risk has two components:

(i) Day-to-day time varying volatility. This can be accurately forecast using the GARCH-dynamic conditional correlation model, proposed by Nobel Prize winner Robert Engle in 2002, combined with appropriately calibrated Ledoit-Wolff shrinkage estimation methods.
(ii) More rare extreme tail realizations. These are controlled for using downside risk measures taking into account the asymmetry, fat tails and extreme dependence of the return distribution (Danielsson, 2011).

A key step prior to the implementation of those advanced estimation methods is to use suitable filters to carefully detect and clean aberrant observations in the data series. A final ingredient to create portfolios that match with investors' preferences for low risk and stable risk adjusted returns is to combine the forward looking low risk objective with industry and country constraints while minimising portfolio turnover and tracking error.

5.1 Finvex R-Equity Engine

In order to implement all the above steps for the construction of properly risk optimised portfolios, Finvex Group has created the R-Equity engine. The engine offers an advanced approach to equity portfolio optimisation that takes the full dependence of return data into account in the selection of a risk optimised portfolio. This method satisfies compatibility with country-sector constraints, as well as tracking error and turnover objectives.

The starting point is a dynamic screening of the universe to remove the stocks that on the basis of a series of individual risk characteristics have no chance of entering the risk-efficient portfolio. This screening has three essential characteristics:

(i) Multiple forward looking risk characteristics that measure the time-varying downside and systematic risks of the assets. These predictive risk analytics capture the three main stylized facts of equity risk described above.
(ii) Robust estimation methods to reduce the impact of aberrant observations on the predictions.
(iii) Methods from operational research are used so that the screening is risk optimised and takes standard portfolio constraints into account. This ensures that the optimised pool continues to show composition characteristics in line with the original universe.

A vital step in portfolio optimisation is to extract accurate estimates on the conditional co-moment matrices describing the dependence between the stock returns. By "conditional co-moments" we mean the predicted value of the co-moment over the next investment period.

It is now well recognised that financial market risks tend to cluster in time, and that it is possible to accurately model these dynamic dependencies. This implies giving higher weights to the more recent observations than to older data. Clearly, those weights cannot be chosen in an ad hoc fashion, but are optimised using maximum likelihood techniques.

For volatility the R-Equity engine applies a variety of GARCH models (Bollerslev, 2008). For correlation, co-skewness and co-kurtosis matrices, the prediction method has to cope with the fact that the number of parameters to estimate can be large compared to the number of time series observations. To allow for this, R-Equities uses so called “structured” estimators, such as Ledoit-Wolff shrinkage and factor models (Ledoit and Wolf, 2003, Martellini and Ziemann, 2009).

Of course, the dependence is also time-varying in a non-linear fashion, e.g. it is well known that correlations tend to increase in bear markets. The methodology deals with this through the use of dynamic conditional correlation models and allowing for time-varying factor exposures (Engle, 2002, 2009).

The final step is then to integrate the conditional co-moment estimates in a portfolio optimiser that can handle multiple non-linear objectives. These include minimum risk, minimum tracking error, minimum turnover, maximum skewness and highest risk diversification.

The optimiser must combine these with flexible constraints on sector-country weights, maximum allowed risk, tracking error and turnover. The potential for such non-convex optimisation problems with non-differentiable objective function requires a global derivative-free optimiser. R-Equity utilizes a differential-evolution based algorithm that is adequate to solve those problems.

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