FINVEX WHITE PAPER ON SMART HARVESTING OF EQUITY STYLE PREMIA

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Executive Summary

Several studies have shown that low risk and value equity investment styles both offer access to a so called factor equity risk 'premium'. But each style appears to perform differently depending on the equity market regime. Therefore, significant diversification opportunities exist across the life cycles of these equity investment styles. A professionally designed style premium diversified portfolio can benefit from the difference in cycles and offer even more consistent returns.

In recent years, some portfolio solutions that combine the low risk and value investment styles in a sequential approach have appeared in the marketplace. The methodology is called sequential as it generally consists in a first stage of the elimination of highest beta stocks, followed by a selection within the remaining sub-universe, based on value criteria.

We propose a superior unified framework to combine low risk and value investing in a single optimal portfolio. Our approach does not treat the two objectives sequentially. We first construct low risk and value investment pools out of one universe. Then the optimization selects stocks from both sub-universes into a resulting portfolio that targets the lowest overall variance. Key is the constraint that low risk stocks and value stocks are targeting to contribute equally to this portfolio's overall variance.

We argue that our approach deals with most of the drawbacks of sequential methods. A sequential approach is by definition suboptimal as it ignores dependence between the low risk and value portfolios. Moreover, beta is time-varying and hence the proportion of excluded high beta stocks should depend on the dispersion of the underlying distribution of betas.

The approach of risk allocation across the low risk and value investment styles we propose is illustrated on a global equity universe.

Introduction

For many years, scholars and investment professionals have argued that a successful implementation of a minimum variance portfolio strategy requires additional constraints. Usually, these constraints are directly imposed on the portfolio weights (Jagannathan and Ma, 2003; Fan et al, 2012) or on the covariance matrix estimate (Ledoit and Wolf, 2003).

An alternative approach that we recommend is to combine single stock weight constraints with a risk allocation constraint on classes of assets. Finvex' white paper of September 2012 already illustrated the performance gains of dynamic risk allocation strategies across bonds and equities.

In this white paper, we aim at illustrating the benefit of risk allocation in a fully invested equity portfolio.

When making equity portfolio allocation decisions, it is common to categorize stocks into broad styles such as value stocks, growth stocks, large-cap stocks, and low risk stocks. The process of setting constraints among the styles is known as "style investing". Note that the process of risk allocation across styles is complicated by the fact that styles do not need to be mutually exclusive. We will focus in this white paper on risk allocation portfolios that combine low volatility and value style investing, but the approach is more generally applicable to include also other styles as 'growth', 'momentum' etc.

Low volatility and value investing are known to be defensive equity investment styles that, on the long run, exhibit superior risk-adjusted returns than the market (Baker and Haugen, 2012, Lakonishok et al., 1994). Several behavioural theories have been put forward that predict that those premiums will continue to exist, albeit with sub-periods of relative underperformance.

The specificity of the life cycle of each investment style is a crucial feature of style investing and this is not different for low risk or value investing. For example, in market rallies the low risk investment style (having a less than unity market beta) is expected to underperform the market. At the same time, underpricing tends to be corrected, leading to superior returns for the value portfolios that are disproportionately invested in undervalued stocks. But if a rally continues for a longer period and investor sentiment reaches exaggerated levels of optimism, the value portfolio could underperform the market, as was the case with the technology bubble in the late 1990s. Finally, if investor sentiment turns back to normal levels, the value portfolio and the low risk portfolio can outperform again the market.

The graph on the following page illustrates the different life cycles of both the low risk and value investment styles. We plot the 1-month return of our proposed value and low risk investing style pools over their selection universe and this over the period 2002-2012.

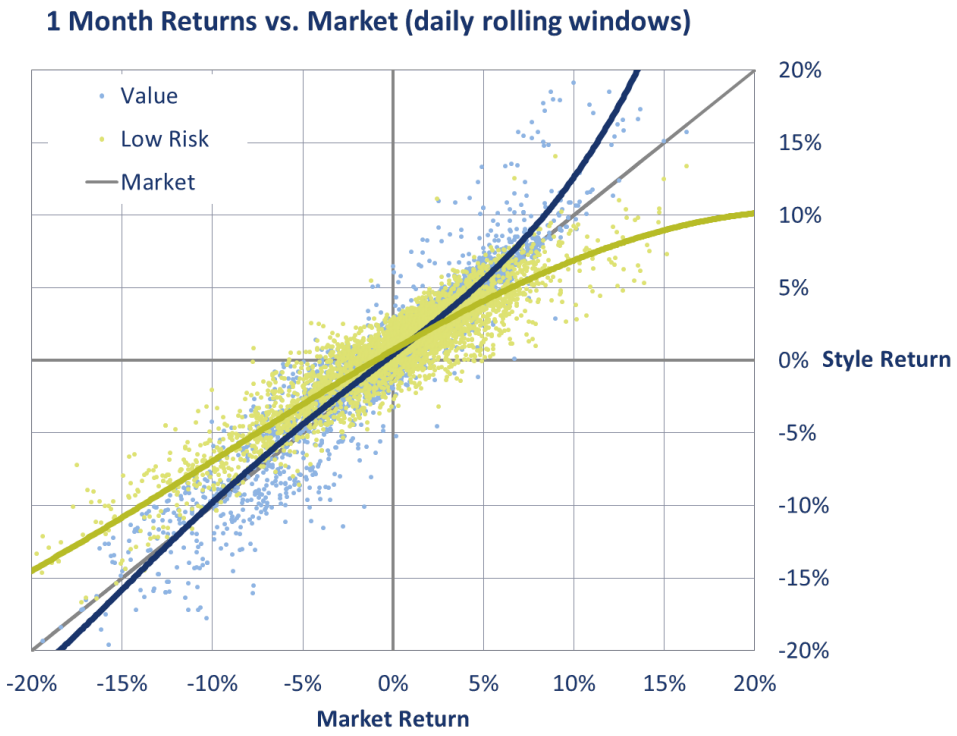

Figure 1 Scatter plot of monthly low risk and value style returns versus the market portfolio return for a global universe over the period February 2002-December 2012.

As can be seen, the diversification opportunities across the life cycle of investment styles are a clear advantage of style investing. By combining styles in a portfolio, it should be possible to reduce the dependence of the portfolio return on the market regime, while preserving an attractive overall risk-adjusted return.

The standard way of such combined style investing is to impose a target weight allocation across styles. Such a constant style mix policy buys the style as the strategy falls and sells it as it rises (see e.g. Perold and Sharpe, 1988). This creates an additional risk as style returns tend to be characterized by positive autocorrelation patterns on the short run (Barberis and Schleifer, 2003).

Recently, portfolio solutions have been launched that take a sequential approach by first excluding a fixed proportion of the highest beta stocks and then weighting the subuniverse based on fundamental accounting ratios.

We argue that such an approach has several major drawbacks. A sequential approach is by definition suboptimal as it ignores dependence between the low risk and value portfolios. Moreover, betas are time-varying and hence the proportion of excluded high beta stocks should depend on the dispersion of the underlying distribution of betas.

We find it more consistent to allocate across investment styles based on the percentage contribution to the portfolio variance of each style and this in the framework of an overall minimum variance portfolio objective. This leads us to buy the style as the style becomes relatively less risky and sell it when its relative risk increases whilst targeting a minimum variance objective that considers the co-variance matrix of all the stocks that belong to one or both of the styles.

In what follows, we first review the main theoretical aspects of value and low risk investing. We then discuss the return properties of value and low risk investing for an equity world universe and illustrate the diversification gains achieved through a risk allocation approach.

Value investing

Value investors buy stocks that they believe are cheap with respect to their fundamentals. This investment paradigm has a long tradition of both scholars and investment professionals arguing that the market can be outperformed by buying stocks that have low prices relative to earnings, dividends, historical prices, book assets or other measures of value.

Several explanations have been given for the existence of a value premium.

There is the group of non-believers that attribute the value premium to data mining (Lo and Mackinlay, 1990) or to a higher systematic risk exposure (Fama and French, 1992).

In recent years, the publications by scepticists are outnumbered by believers who attribute the value premium to the contrarian character of value investing. Compared to the market portfolio, value portfolios tend to invest disproportionately in stocks that are underpriced and underinvest in stocks that are overpriced.

Several theories have been proposed to explain the mechanisms behind the 'mispricing' of stocks. Lakonishok et al (1994, 1997) attribute the value premium to the existence of naive investors who extrapolate past earnings growth too far into the future or overreact to good or bad news. Barber and Odean (1998) argue that individual investors are net buyers of attention-grabbing stocks and thus an increase in individual investor attention results in temporary positive price pressure. A more agnostic view is to recognize the presence of liquidity or noise traders (motivated by different reasons than the theoretical CAPM investor such as taxes, fiduciary responsibilities, portfolio rebalancing or personal reasons). Under this noisy market hypothesis, stock prices have a temporary mispricing (Arnott et al, 2005, Treynor 2005).

The relative performance of value stocks goes through cycles of over –and underperformance with respect to the market. The tech rally in the late 1990s and the quant meltdown in 2007 are two episodes when value stocks experienced periods of relative decline (Owyong, 2001).

The life cycle of relative performance of value investing is closely connected to investor sentiment. In case of excessive optimism or pessimism, investors evaluate incorrectly asset values, causing asset prices to deviate from their intrinsic values. The mispricing gets corrected as the economic fundamentals are revealed and sentiment wanes. Chan and Lakonishok (2004) document, e.g., that growth stocks rocketed in the late 1990s because investor sentiment reached exaggerated levels of optimism about the prospects for technology, media, and telecommunications stocks.

Through a regression analysis, Owyong (2011) shows that high values of the implied volatility index (VIX) reduce the performance of the value strategy, ceteris paribus. This calls for a portfolio approach whereby the value strategy is diversified with a strategy that can be expected to perform better in

periods of high uncertainty. Because of the flatness of the risk-return relationship, the low risk investing style acts in our view as such a diversifier.

Low risk investing

The family of low risk investing styles groups simple rules such as 1/beta weighting and more sophisticated constrained portfolio risk minimization strategies. Several authors have documented that, over long time periods, low risk portfolios outperform high risk portfolios and the market capitalization weighted portfolio.

E.g. Clarke, de Silva and Thorley (2006) find that minimum variance portfolios based on the 1000 largest U.S. stocks over the 1968-2005 period achieve a volatility reduction of about 25% while delivering comparable or even higher average returns than the market portfolio. A recent comprehensive study by Baker and Haugen (2012) confirmed the finding that the risk-return relationship for stocks is extremely flat, and that investing in low risk stocks yields higher risk adjusted returns than investing in low risk stocks.

There are several behavioural reasons for the underpricing of low risk stocks: investors are attracted to high risk "lottery" tickets stocks, institutional investors are benchmarked with respect to market capitalization indices and have limits on how much leverage they can take and therefore overweight high beta stocks (Baker et al, 2011).

A further fundamental advantage of taking a low risk portfolio objective is that the compounded return of a low volatility strategy will be higher than the return of a high volatility strategy with the same arithmetic average return. See Boudt and Peeters, Finvex White Paper (April 2012) for more details on the low volatility anomaly.

Like the value strategy, the low risk strategy has a lifecycle of periods of relative outperformance alternating with periods of relative underperformance. The lifecycle is directly connected to the fact that low risk strategies have a less than one beta. As such, they underperform in bull markets and outperform in bear markets. But over the complete lifecycle, the low risk strategy is characterized by higher risk adjusted returns, as explained above.

Proposed Methodology

We consider an equity universe of *N* stocks, with covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1N}\sigma_1\sigma_N \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \cdots & \rho_{2N}\sigma_1\sigma_2 \\ & & \ddots & \vdots \\ \rho_{1N}\sigma_1\sigma_N & \rho_{2N}\sigma_1\sigma_2 & \cdots & \sigma_N^2 \end{pmatrix},$$

with ρ_{ij} the correlation between stock *i* and *j*, and $\sigma=(\sigma_1,\ldots, \sigma_N)'$ the vector of individual volatilities of the *N* stocks.

For a portfolio with weights $w=(w_1,\ldots, w_N)'$, the portfolio standard deviation is given by

$$\sigma(w) = \sqrt{w'\Sigma w}.$$

An important property of the portfolio standard deviation is its sub-additivity, meaning that, because of diversification effects, the portfolio standard deviation is always less or equal than the weighted average volatility:

$$\sigma(w) \leq w'\sigma.$$

The percentage volatility risk contribution of the *i*th asset in the portfolio is given by:

$$\%RC_i = w_i (\Sigma w)_i / w'\Sigma w.$$

The global portfolio objective is to minimize the portfolio variance balancing simultaneously the volatility and correlation diversification properties of the stocks:

$$w^{MV} = \arg\min w'\Sigma w.$$

We re-estimate the covariance matrix for each monthly selection date using the covariance matrix based on the dynamic conditional correlation model of Engle (2002), together with correlation targeting towards a structured unconditional correlation matrix. Without constraints, the minimum variance portfolio is too concentrated in a few assets and sectors. We therefore impose additionally a cardinality constraint of 250 stocks, as well as lower and upper weight constraints that depend on the stock's market capitalization.

Let I_i and J_i be 0/1 indicators for asset i being a value and low risk stock, respectively. Note that, for stocks that are low risk and high value, we have that the product of I_i and J_i is 1, and 0 otherwise. We decided to allocate the risk contribution of such a duplicate stock equally across the two investment styles. The equal-risk allocation constraints across the value and low risk stocks are the following:

$$0.5 - \varepsilon \le \sum_{i=1}^{N} \%RC_i \, I_i \, (1 - J_i) + \frac{1}{2} \sum_{i=1}^{N} \%RC_i \, I_i \, J_i \le 0.5 + \varepsilon$$

$$0.5 - \varepsilon \le \sum_{i=1}^{N} \%RC_i \, J_i \, (1 - I_i) + \frac{1}{2} \sum_{i=1}^{N} \%RC_i \, I_i \, J_i \le 0.5 + \varepsilon,$$

with ε=0.025 a tuning constant to avoid numerical problems due to setting too tight the risk allocation constraint (a nonlinear function of the portfolio weights and all elements of the covariance matrix).

Finally, a turnover constraint is imposed, and a heuristic optimisation algorithm is used to find the solution to this non-linearly constrained quadratic objective optimisation problem.

Diversification gains in combining value and low risk investing

We consider now a global stock universe and construct for each month since May 2002 a pool of 250 'value' stocks and 250 'low risk' stocks. These universes are constructed based on measuring the value and low risk characteristic of stocks using a proprietary composite score methodology together with an optimisation algorithm ensuring sector and country diversification within each selection pool, as well as liquidity constraints.

The red and blue lines in Figure 2 show the evolution of the cumulative outperformance of an equally-weighted investment in the value and low risk investment pools, relatively to cumulative value of the market.

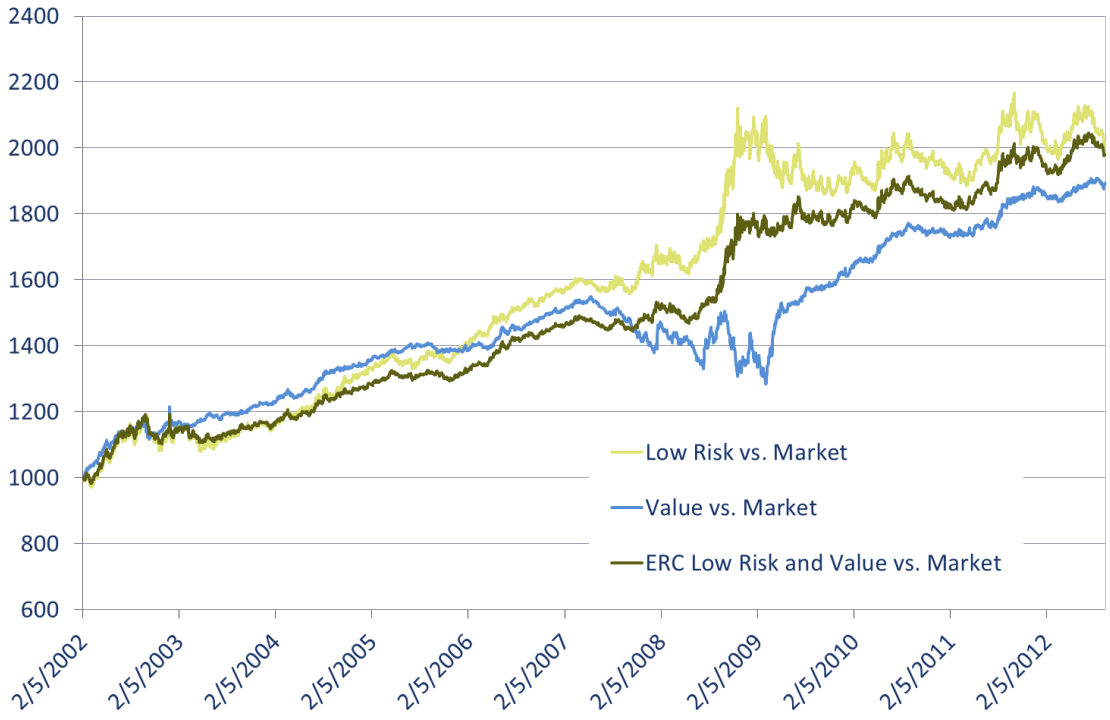

Figure 2 Relative cumulative value of low risk, value and equal risk contribution low risk-value portfolio versus the market portfolio for a global universe over the period February 2002-December 2012.

We can make two important observations. First, over the complete 2002-2012 period, the value and low risk universes outperform the market by more than 50%.

Second, the life cycles of relative performance of the two investment styles are significantly different.

The low risk universe outperformed the market especially in the period 2002-Feb 2009, underperformed during the market rally of March-April 2009 and since then has had a similar performance as the market. In contrast, the value strategy outperformed the market in the period 2002-May 2007 and April 2009-present. It underperformed during the quant meltdown in April 2008-March 2009.

The empirical analysis on this global universe confirms the existence of a low risk and value premium in a global equity universe and the diversification possibilities across the lifecycles of the value and low risk investment styles.

To exploit these empirical facts, we use the methodology described above to construct the portfolio with the lowest possible variance, under the constraint that low risk and value stocks contribute equally to the portfolio variance. Besides such an equal risk contribution constraint, we imposed that the portfolio is invested in 250 stocks and that the maximum weight allowed for each stock is constrained by its market capitalization.

Figure 3 shows the percentage risk contribution (upper plot) and weight allocation (lower plot) of the low risk and value stocks, disentangling the three sets of stocks: low risk stocks that are not classified as value stocks, value stocks that have not the low risk property and stocks that have both the low risk and value characteristic.

By definition, the equal risk contribution constraint implies an increase in the weight of low risk stocks, when the relative risk of value stocks increases. As can be seen in Figure 3, this has happened during the financial crisis in 2008, where the weight allocation of the low risk stocks increased to over 60%.

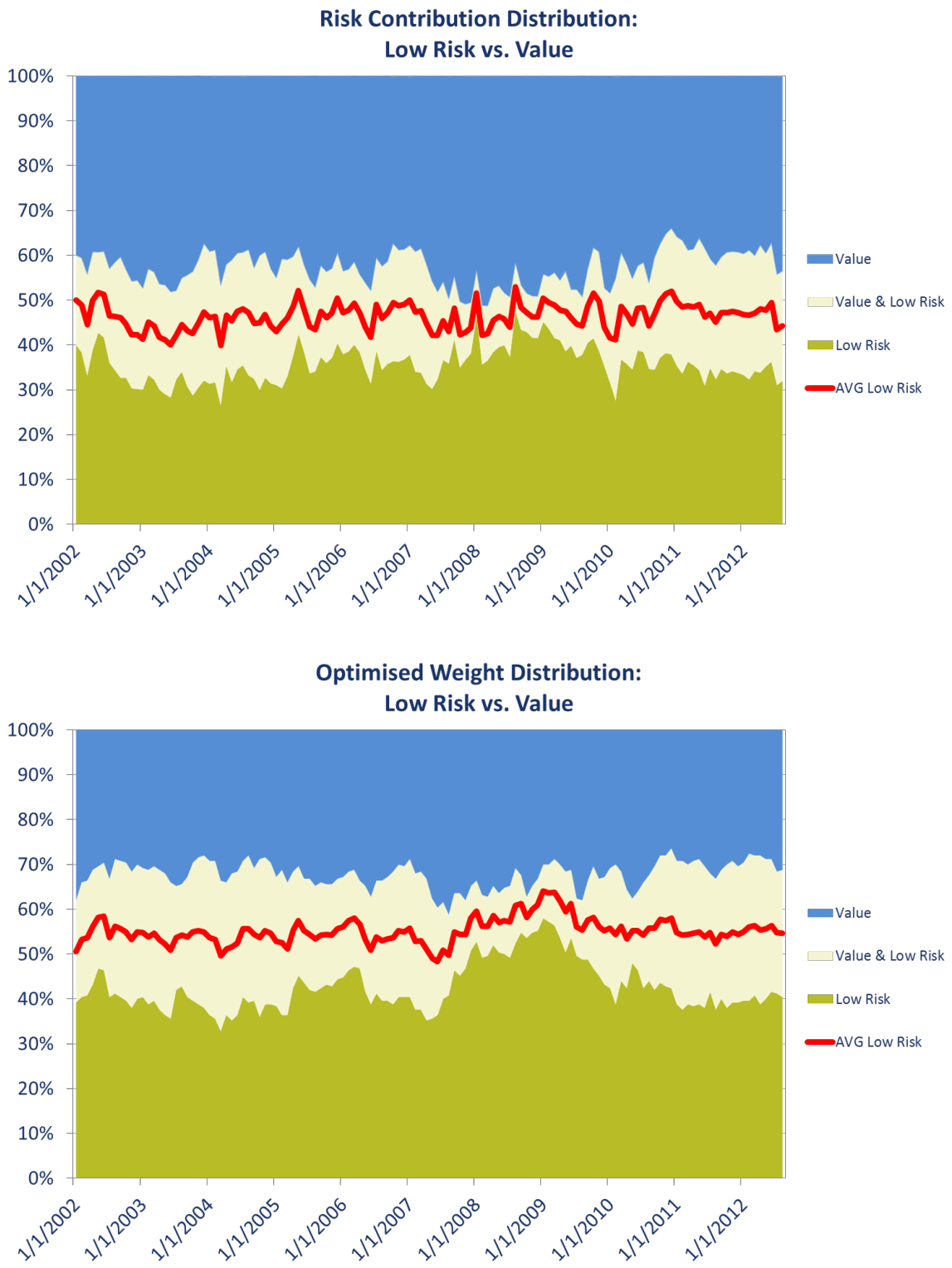

Figure 3 Percentage risk contribution (top panel) and weight allocation (bottom panel) of the stocks included in the equal-risk-contribution value and low risk portfolio. Stocks are grouped in 3 sets: low risk stocks that are not a value stock, value stocks that are not low risk stocks, stocks that have the value and low risk characteristic. The rebalancing is monthly and the portfolio weights and percentage risk contributions are shown for the period February 2002-December 2012.

The green line in Figure 2 shows the relative value of this optimised equal-risk-contribution low risk and value portfolio.

Note that the optimised portfolio outperforms the market by 60% over the 2002-2012 period. By combining the value and low risk styles into one single portfolio, the individual life cycle of each strategy is diversified away into 'stable' outperformance.

These results are confirmed also by the return analysis in Table 1, where we see that the value and low risk pools (equally weighted stocks) and the equal risk contribution minimum variance portfolio have an annualised return of 5.1%, 5.9% and 5.7% respectively, substantially higher than the annualised return of the market cap weighted portfolio of 1%. The volatility of the low risk pool (11%) and the equal risk contribution portfolio (12%) is significantly lower than the volatility of the value pool (16%) and of the market cap weighted portfolio (15%). The value pool (58%) and market cap weighted portfolio (49%) have large drawdowns compared to the low risk pool (37%) and the equal risk contribution portfolio (41%).

The second panel of Table 1 illustrates further the differences in performance of the investment styles in function of the market regime. The low risk portfolio has smaller losses in bear markets and smaller gains in bull markets than the market portfolio. The value strategy has higher gains in the bull months and relatively large losses in the bear market months. Combining the low risk and value stocks through an equal risk contribution approach leads to a portfolio with smaller losses in bear markets and an attractive upside potential in rising markets.

Table 1 Monthly returns of the market cap weighted, value equally weighted, low risk equally weighted and equal risk contribution low risk-value portfolios invested in the DJSIE 1200 universe.

	Annualized return	Annualized standard deviation	Sharpe ratio (RF=0)	Max drawdown
2002-2012				
S&P 1200 market cap weighted index	0.012	0.149	0.081	-0.489
Equally weighted value subuniverse	0.051	0.162	0.315	-0.576
Equally weighted low risk subuniverse	0.059	0.112	0.528	-0.367
Equal risk contribution value-low risk portfolio	0.057	0.121	0.471	-0.407
2002-2012 – Months with Negative Market Return				
S&P 1200 market cap weighted index	-0.142			
Equally weighted value subuniverse	-0.130			
Equally weighted low risk subuniverse	-0.083			
Equal risk contribution value-low risk portfolio	-0.096			
2002-2012 – Months with Positive Market Return				
S&P 1200 market cap weighted index	0.420			
Equally weighted value subuniverse	0.470			
Equally weighted low risk subuniverse	0.330			
Equal risk contribution value-low risk portfolio	0.057			

Conclusion

Low volatility and value investing are known to be defensive equity investment styles that, on the long run, exhibit superior risk-adjusted returns than the market. The specificity of the life cycle of both investment styles is a crucial feature and offers diversification opportunities. By combining styles in a portfolio, it is possible to reduce the dependence of the portfolio return on the market regime, while preserving an attractive overall risk-adjusted return.

The standard way of such combined style investing with target weight allocation across styles creates an additional risk as style returns tend to be characterized by positive autocorrelation patterns on the short run (Barberis and Schleifer, 2003). Portfolio solutions that take a sequential approach have several major drawbacks as they ignore dependence between the low risk and value portfolios and time-variation of beta.

We propose to allocate across both investment styles based on the percentage contribution to the portfolio variance of each style and this in the framework of an overall minimum variance portfolio objective. Our analysis shows that, by combining the value and low risk styles into a smart single portfolio, the individual life cycle of each strategy is diversified away into 'stable' outperformance.

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