FINVEX

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The Margrabe Best-of-two strategy:

A sweet spot between equity upside potential and resilience to equity downturns

Executive Summary

Medium to long-term investors are faced with the cyclical behavior of equity markets. Portfolio protection techniques help the investor to achieve a stable performance that on the medium to long run is similar to the equity market performance, but avoid the large drawdowns by timely shifting to a low risk asset.

The Margrabe Best-of-two portfolio allocation offers such portfolio protection. When applied to equities and bonds, it invests, at the year-turn, equally in equities and bonds. It then uses option-based theories to progressively invest more in the best performing asset. Thanks to the underlying option theory, its performance benefits from the upside potential of investing in the best performing asset, and the downside protection of holding a substantial portion of the portfolio invested in bonds. It is fully rule-based and thus excludes behavioral biases.

To illustrate the effectiveness of the Margrabe Best-of-two strategy, let us consider the case of investing in a European equity and bond index from Feb-2007 to Feb-2017. For the bond index, we take the Merrill Lynch European Government bond index. As equity index, we compare the results for the Finvex Sustainable & Efficient Europe equity index (FSEE) with the traditional Dow Jones Sustainability Europe index (DJSE). All of them are total return indices, in EUR. The two equity indices track the performance of sustainable EU stocks. The main difference between the DJSE and the Finvex Sustainable & Efficient Europe index is that the DJSE is a not optimized (it uses market capitalization weighting), while the Finvex index does thorough screening to invest only in low risk stocks.

Performance	Equity indices		Bond index	Margrabe Best-of-two	
	DJSE	Finvex		DJSE+Bond	Finvex+Bond
Cumulative value (€1)	1.27	2.17	1.63	1.76	2.30
Annualized return (%)	2.43	7.98	4.94	5.75	8.61
Annualized volatility (%)	15.65	12.12	4.44	8.21	7.97
Sharpe ratio	0.16	0.66	1.11	0.70	1.08
Maximum drawdown (%)	55.84	39.38	5.76	11.89	11.16
95% Modified VaR (%)	7.43	5.43	1.71	3.54	3.07

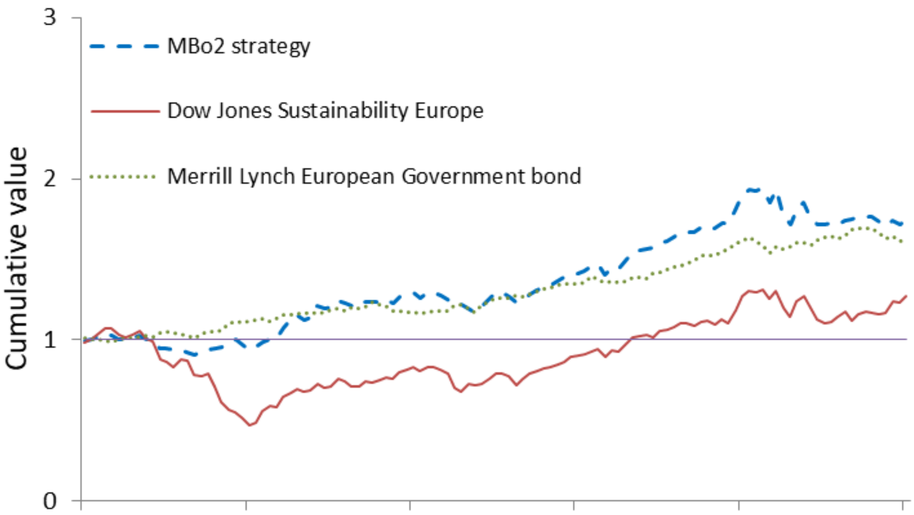
Note: DJSE: Dow Jones Sustainability Europe Index; Finvex: Finvex Sustainable & Efficient Europe.

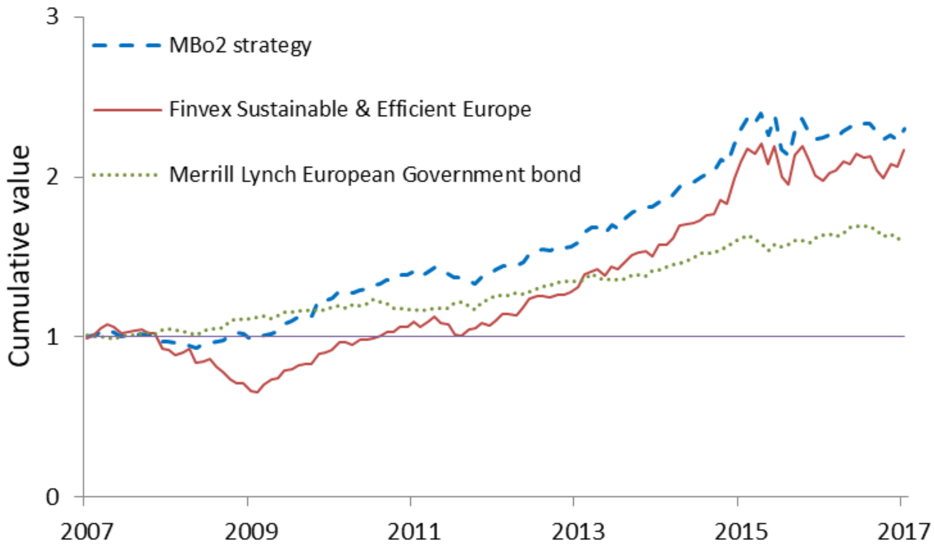
This difference in market capitalization versus low risk weighting has material effects in terms of performance on the decade of returns analyzed. Over the period 2007-2017, the Finvex equity index outperforms the DJSE index in all dimensions: a higher annualized return (7.98% vs 2.43%), a lower volatility (12% versus 16%), and a lower 95% VaR (5.43% against 7.43%).

The Finvex equity index also has a higher resilience to market downturns. Its maximum drawdown is 40%, compared to the 56% of the DJSE. Such a level of drawdown is inherent to a pure equities investment product. A capital protection overlay to the Finvex equity index reduces further the drawdown.

In the right panel of the performance table above we investigate the use of the Margrabe Best-of-two strategy, which uses option-based theories to optimally allocate between equities and bonds. We find that the dynamic allocation using the Margrabe strategy leads to a drawdown of only 11%, while average performance when implemented with the Finvex equity index is still excellent: 8.61% annualized return and a volatility of 8%.

The gain in stability in performance becomes even more clear when considering the cumulative performance charts. The Magrabe Best-of-two strategy avoids the large equity drawdowns of the financial crisis, and, compared to the bond index, benefits from the high upside potential of investing in the Finvex Sustainable & Efficient Europe equity index.





Over the past months, researchers at the Finvex Quantitative Strategies team have investigated in detail the sources of performance. They appreciate in particular the elegance of the underlying option-based theories in combining relative performance, volatility and correlation when constructing the optimized portfolio with a stable performance profile. Their results are in the attached research paper.

I wish you a pleasant read and please don't hesitate to contact us for any further inquiries on designing tailor-made asset allocation strategies.

Kind regards,

Stefan Hartmann,

Head of Quantitative Research, Finvex

Authors

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Properties of the Margrabe Best–of–two strategy to tactical asset allocation☆

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Abstract

The Margrabe Best–of–two (MBo2) strategy is a rule–based dynamic investment solution for the two–asset allocation problem. Its typical implementation involves yearly rebalancing the portfolio weights to 50–50 between a high–risk and low–risk asset. It uses intra–year weight adjustments to chase the momentum of the best performing asset by replicating the value of a Margrabe option to exchange an asset for another asset. In practice, this means that the Margrabe portfolio allocation benefits from the upside potential of the high–risk asset and the downside protection from the low–risk asset. The MBo2 allocation depends on the assets' prices, their return volatilities and correlation, as well as the remaining time until year–end. In this paper, we derive analytical formulae and use simulations to provide insights on the sensitivity of the strategy's weights and performance to these input parameters. We also report the results of an extensive out–of–sample evaluation for the bond–equity investment problem.

Keywords: Best–of–two, Bond–equity, Margrabe, Tactical asset allocation, Upside potential, Downside protection.

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1. Introduction

Bond–equity, equity–gold, domestic equity–emerging markets equity; these are all important two–asset allocation problems. Ideally, the tactical asset allocation is such that the portfolio performs nearly as well as what *ex post* turns out to be the best performing asset. This objective is inherent in a tactical asset allocation that uses the Margrabe Best–of–two (MBo2) strategy. Commercial examples include the NYUSDA index (see, *e.g.*, Kula et al., 2017) and Metzler best–of–Germany index, among others. Their investment rule consists of dynamically allocating the portfolio between the high–risk asset (the NYSE U.S. Large Cap Equal Weight Index (NYLGCAPT) and German large cap equity future index) and the low–risk asset (NYSE Current 10–Year U.S. Treasury Index (AXTEN) and German bond future). The index uses monthly rebalancing. At the end of the year, the portfolio makes a reversal trade by setting the weight allocation to an equal 50–50 investment in the high– and low–risk assets. Within the year, a momentum strategy is pursued by setting weights such that they replicate the value of the Margrabe option to exchange the low–risk asset against the high–risk asset.

An unresolved question is to understand how the drivers of the Margrabe option value interact in determining the MBo2 portfolio weights and performance. We investigate this for both backward–looking variables, like the relative price of the high versus low–risk asset, and forward–looking parameters, like the high– and low–risk assets' return volatilities and correlation.[1] We derive explicit formulae for the marginal impact of the input parameters on the portfolio weights, and use numerical experiments to evaluate the impact in case of large changes in the parameters. We then apply historical simulation to investigate the drivers of the performance of the MBo2 strategy over the one– and five–year horizons. In particular, we show that the relative price of the high–risk asset over the low–risk asset and the volatility of the high–risk asset's return are the primary drivers of the portfolio composition and portfolio performance of the MBo2 strategy. The correlation between two assets' returns and the volatility of the low–risk asset's return are less influential. We document the upside potential and downside protection properties of the strategy using real–world and block–bootstrap simulated data.

In addition to analyzing the determinants of the portfolio allocation and performance, we propose several modifications to the traditional implementation of the MBo2 strategy. They include the use of option-implied volatility of the high–risk asset's return, a change in the reset date, and a different definition of the maturity of the exchange option, among others. We find that the risk-adjusted performance improves especially when calibrating the volatility using option-implied volatility rather than the sample or GARCH–based volatility. It also leads to lower turnover and drawdowns.

The paper is organized as follows. We first present the MBo2 strategy in Section 2. Section 3 studies the sensitivity of the MBo2 allocation to the input parameters. The simulation study of the MBo2 allocation's performance determinants is presented in Section 4. We propose and test alternative implementations of the MBo2 strategy in Section 5. Section 6 concludes. Proofs of the various derivations are presented in the Appendices.

2. The MBo2 strategy

2.1. Definition

We consider the problem of constructing a portfolio invested in a relatively high–risk asset (such as an equity or a portfolio of equities) and a relatively low–risk asset (such as a government bond or a portfolio of

[1] In this paper, we use the logarithmic returns of assets' prices to calculate assets' return volatilities and correlation. For portfolio performance, the simple returns of assets' prices are applied.

bonds). These assets are denoted by A and B, respectively. Their public prices at month–end t are denoted by $P_{A,t}$ and $P_{B,t}$.

The Margrabe allocation strategy involves investment decisions at two frequencies. At a low frequency (typically yearly), it sets the weights of the two assets to 50%. At a higher frequency (typically monthly), the portfolio is rebalanced to replicate the value of the Margrabe option to exchange an asset for another asset. Henceforth, we call the low–frequency rebalancing dates as base dates. Without loss of generality, we assume to have only two base dates, namely t_0 and t_1, and suppose that the high–frequency rebalancing dates are denoted by t, with $t_0 < t < t_1$.

The European option to exchange an asset for another at a maturity date is computed by Margrabe (1978) under the standard Black–Scholes' assumptions. Before formally defining those weights, we need some more notation. We denote by T the initial investment horizon of the option, defined as the period between the two base dates, t_0 and t_1 ($t_1 = t_0 + T$). Let $\tau \equiv T - t$ be the time to maturity of the option. We denote by $\Phi(\cdot)$ the standard normal cumulative distribution function. Let σ_A and σ_B be the annualized volatility of asset A's and B's logarithmic returns and $\rho_{A,B}$ the correlation between asset A's and B's returns. We assume that σ_A, σ_B and $\rho_{A,B}$ are constant during the remaining investment horizon. We use $\sigma_{A-B} \equiv \sqrt{\sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho_{A,B}}$ to denote the volatility of the logarithmic return of the relative price.[2]

Finally, we denote by $P_{A,t|t_0} \equiv P_{A,t}/P_{A,t_0}$ and $P_{B,t|t_0} \equiv P_{B,t}/P_{B,t_0}$ their cumulative performance since the reset date t_0, respectively. We define the relative price of asset A versus asset B at time t as the relative cumulative performance since the reset date t_0:

$$P_{A/B,t|t_0} \equiv \frac{P_{A,t|t_0}}{P_{B,t|t_0}} .$$

Margrabe (1978) shows that, under the assumptions of Black and Scholes (1973), the price of the option to exchange asset B for asset A at time t is then given by:

$$C_t^{B\to A} \equiv P_{A,t|t_0}\Phi(d_1) - P_{B,t|t_0}\Phi(d_2) , \tag{1}$$

where:

$$d_1 \equiv \frac{\ln(P_{A/B,t|t_0}) + \frac{1}{2}\sigma_{A-B}^2\tau}{\sigma_{A-B}\sqrt{\tau}} \quad \text{and} \quad d_2 \equiv d_1 - \sigma_{A-B}\sqrt{\tau} .$$

Note that the price is an increasing function of the relative price, $P_{A/B,t|t_0}$, the volatility of the logarithmic return of the relative price, σ_{A-B}, and the time until the maturity of the option, τ.[3]

The value of the MBo2 replicating portfolio is the sum of the value of asset B and the option to exchange asset B for asset A. It is given by:

$$P_{\text{MBo2},t} = P_{A,t|t_0}\Phi(d_1) + P_{B,t|t_0}(1 - \Phi(d_2)) . \tag{2}$$

In Appendix A, we show that this is equivalent to computing $P_{\text{MBo2},t}$ as the sum of the value of asset A and the option to exchange asset A for asset B, and thus that $P_{B,t|t_0} + C_t^{B\to A} = P_{A,t|t_0} + C_t^{A\to B}$.

[2] To simplify notation, we omit the time index t in the volatility and correlation parameters.

[3] The latter two become clear by considering the special case where $P_{A,t|t_0} = P_{B,t|t_0}$. Then the option price equals $P_{A,t|t_0}(2\Phi(\frac{1}{2}\sigma_{A-B}\sqrt{\tau}) - 1)$, which is increasing in σ_{A-B} and τ.

From (2), we obtain directly the portfolio weight that replicates the value of the MBo2 portfolio, namely:

$$w^{\text{MBo2}}_{A,t} \equiv \frac{P_{A,t|t_0}\Phi(d_1)}{P_{\text{MBo2},t}} \quad \text{and} \quad w^{\text{MBo2}}_{B,t} \equiv \frac{P_{B,t|t_0}(1-\Phi(d_2))}{P_{\text{MBo2},t}} .$$

In Appendix B, we show that, under the standard Black–Scholes' assumptions, it is equivalent to calculate the weight of the high–risk asset (asset A) in the MBo2 replicating portfolio as:

$$w_{A,t} = \frac{P_{A/B,t|t_0}\Phi(d_1)}{P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))} = \frac{\mathbb{E}[P_{A/B,T|t}|P_{A/B,T|t} > 1]}{\mathbb{E}[P_{A/B,T|t}|P_{A/B,T|t} > 1] + \mathbb{P}[P_{A/B,T|t} < 1]} . \tag{3}$$

The high–risk asset weight in the replicating portfolio then equals the ratio between its expected relative price, when it exceeds one, and the sum of that value and the probability that the relative price ends lower than one at maturity.

Since the portfolio is fully invested, the weight of the low–risk asset (asset B) in the portfolio is $w_{B,t} \equiv 1 - w_{A,t}$. By construction, all weights are in the range of $[0,1]$. In Appendix C, we show that, in the special case where $P_{A,t|t_0} = P_{B,t|t_0}$ (and thus $P_{A/B,t|t_0} = 1$), the weight of the high–risk asset is 50% irrespective of the values for the other parameters. On each of the low–frequency base dates, t_0 and t_1, we thus have $w_{A,t} = 0.5$. Between the rebalancing dates, the weights automatically adjust in function of the relative performance of the assets A and B.

2.2. *Illustration of the MBo2 strategy*

The MBo2 strategy leads to a tactical allocation that, when compared to buy–and–hold investments in one of the underlying assets, has the advantage of benefitting from the upside potential of the high–risk asset and from the downside protection of the low–risk asset. We illustrate this here in the case of high and low–risk assets chosen for investment in the US and Germany. The choice for these markets is inspired by the NYUSDA index and Metzler best–of–Germany tactical asset allocation solutions. The two investment problems that we study are the following:

- Allocate between US equities (S&P500 total return index, in USD) and US bonds (Barclays US Treasury 7-10 year total return index, in USD);
- Allocate between German equities (DAX total return index, in EUR) and German bonds (Germany Treasury 7-10 year total return index, in EUR).

For each of these universes, we compare the performance of the MBo2 with two traditional tactical allocation methods: (i) buy–and–hold investments in one underlying, and (ii) monthly rebalanced constant–mix portfolios.[4] We consider the performance over the period ranging from January 1992 to March 2017 (303 monthly observations).

Implementation. For this practical illustration, we have more than one reset–date. We denote these dates by $t_0 \in \{t_{0,1}, t_{0,2} \ldots t_{0,K}\}$ where the distance between any two consecutive dates is exactly equal to the investment horizon of the option (T). In the base model, the base date is the last trading day of each year.

[4] The market risk–free rate in the US is the T–bill one–month yields collected from the K. French data library: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. The risk–free rate in Germany is the three–month interbank yields, available at the website of the Federal Reserve Bank of St. Louis: `https://fred.stlouisfed.org/`.

On these base dates, the relative price is set at one and the weights on equity and bond are set to 50%/50%. Volatilities and correlation of two assets' returns are estimated using a rolling window of three years of monthly returns (36 observations). The estimated values on a base date $t_{0,k}$ are kept constant during the investment horizon of the option which is assumed to be 12 months ($T = 12$) until the next base date $t_{0,k+1}$. The portfolio weights are rebalanced on a monthly basis with the update of the relative price and the time to maturity of the option. The out–of–sample window is from February 1995 to March 2017.

Impact on the cumulative performance. Figure 1 displays the cumulative performance of the buy–and–hold strategies together with the MBo2 strategy for the US (the top panel) and German market (the bottom panel). For both US and German markets, we see that the cumulative values of the buy–and–hold strategy on the low–risk asset are more stable than those invested in the high–risk asset. This comes at the price of a lower performance. Their cumulative values are half of those of strategies investing in the high–risk asset (4.0 compared with 6.0-8.0). Regarding the MBo2 strategy, we see that volatility is between the values of the buy–and–hold strategy on the high–risk asset and low–risk asset, respectively. It outperforms in terms of a higher end–of–period cumulative value (10.0-11.0) and is more stable, as it avoids large losses in the bearish periods of 2000–2002 (the dot–com crisis) and 2007-2008 (the global financial crisis), and participated in the upside potential of equities in the bullish period.

Impact on the risk–return. In order to gauge the risk–return properties of these investment solutions, we compare in Table 1 the out–of–sample performance statistics of the MBo2 strategy with those of the buy–and–hold strategy on each asset, and the constant–mix strategies of 50/50 equal-weighting and 60/40 weighting.

In Panel A of Table 1, we find that, for the US market, the MBo2 strategy has the highest annualized return (10.69%) and the highest Sharpe ratio (0.77) over the out–of–sample evaluation period February 1995–March 2017. Its volatility is lower than the buy–and–hold strategy on the high–risk asset and the constant–mix 60/40 strategy (8.52% versus 14.82% and 8.72%). The buy–and–hold strategy on the bond index has the lowest values of volatility and drawdown (6.26% and 7.37%, respectively). The lower risk comes at the cost of a lower return (6.24%). The reverse applies to the buy–and–hold strategy on the equity index. The equally–weighted or constant–mix 60/40 strategies generate returns, volatilities, and drawdowns in the range of the buy–and–hold strategy on equities and bonds. It is interesting to note that the worst drawdown of the MBo2 strategy (14.3%) was in August 1998 when the market crashed by 14.6% in a month. Meanwhile, during the financial crisis of 2008, the MBo2 strategy only suffered from a drawdown of 10.0% compared with 51% for the buy–and–hold strategy on the equity. This illustrates the downside risk protection offered by the MBo2 strategy compared to buy-and-hold investment.

As can be expected, the monthly rebalancing to replicate the value of the Best–of–two portfolio and the yearly rebalancing to the 50–50 constant mix leads to a relatively high annualized turnover of 62.53%. To show that the outperformance in terms of higher Sharpe ratio is also to be expected in terms of net returns (after transaction costs), we report in the last column of Table 1 the so–called break–even transaction costs (BETC) of the MBo2 versus the four alternative investment strategies. The BETC is defined as the fee (expressed in cents per dollar traded) that the MBo2 strategy can charge such that it has, in net returns, an equal Sharpe ratio with the alternative strategy. As can be seen in Table 1, it varies between 86 and 178 cents per dollar traded. We can thus conclude that the outperformance of the MBo2 strategy in terms of Sharpe ratio is high enough to be robust to realistic values of transaction cost.

Similar results are observed when investing in the German market, where the MBo2 strategy yields the highest return, and a lower volatility and drawdown than the buy–and–hold strategy on equities.

Figure 1: Cumulative values of strategies on two–asset allocation
The figure displays the out–of–sample cumulative values of the buy–and–hold and the MBo2 investment in the high–risk asset (S&P 500 total return index or DAX total return index, the full black line) and the low–risk asset (Barclays US Treasury 7-10 year total return index or Germany Treasury 7-10 year total return index, the gray dotted line) over the period February 1995 to March 2017. Top (resp. bottom) chart shows the cumulative value of $1 (resp. €1) invested in the U.S (resp. German) market. See Section 2.2 for details.

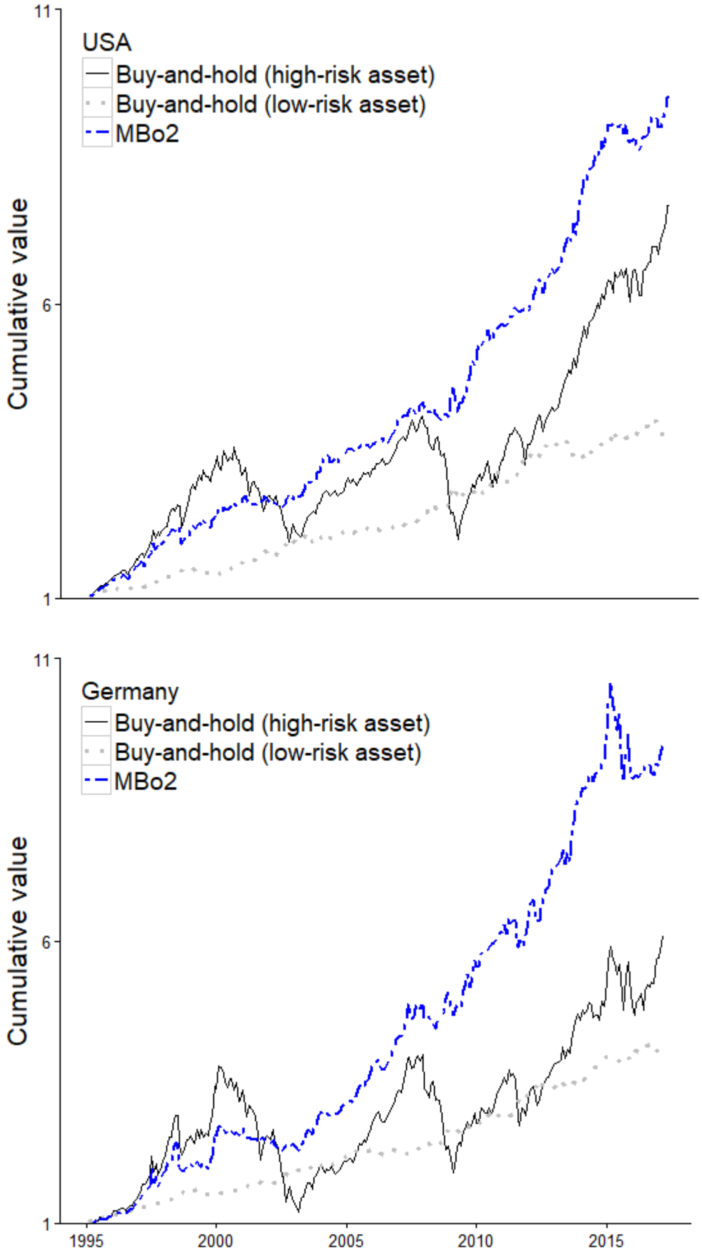


Table 1: Performance of tactical allocation strategies in equities and bonds for the US and German markets
This table presents the portfolio performance of the buy–and–hold strategy for each asset, the constant–mix strategies (Equally weighted – EW, constant–mix of 60% equity and 40% bond – CM 60/40), and the MBo2 strategy (MBo2). For each strategy, seven performance criteria are presented: Cumulative value of $1 investment (CVal), annualized geometric return (Mean, in percent), annualized standard deviation (Std, in percent), annualized Sharpe ratio (SR), maximum drawdown (MaxDD, in percent), 5% modified Value-at-Risk (MVaR, in percent), the average annualized turnover (TO, in percent), and the break–even transaction cost that makes the annualized Sharpe ratio of the MBo2 strategy equal to the annualized Sharpe ratio of the alternative strategy with lower Sharpe ratio computed on gross returns (BETC, in percent). The out–of–sample period ranges from February 1995 to March 2017, for a total of 266 monthly observations. See Section 2.2 for details.

Strategies	CVal	Mean	Std	SR	MaxDD	MVaR	TO	BETC
Panel A: Tactical allocation strategies for the US market								
Equity	7.68	9.64	14.82	0.37	50.95	6.85	–	1.78
Bond	3.82	6.24	6.26	0.36	7.37	2.37	–	1.43
EW	5.89	8.33	7.41	0.57	23.28	3.08	6.14	0.86
CM 60/40	6.30	8.66	8.72	0.52	29.69	3.75	5.91	1.14
MBo2	9.51	10.69	8.52	0.77	14.30	3.35	62.53	–
Panel B: Tactical allocation strategies for the German market								
Equity	6.08	8.48	21.49	0.23	68.29	9.98	–	2.50
Bond	4.06	6.52	4.79	0.64	7.61	1.75	–	–
EW	5.79	8.24	10.38	0.45	33.56	4.45	8.31	1.23
CM 60/40	6.00	8.42	12.53	0.39	42.20	5.52	8.03	1.67
MBo2	9.48	10.68	12.00	0.59	20.53	4.49	57.34	–

3. Drivers of the weight allocation of the MBo2 strategy

From the weight definition in (3), it follows that the MBo2 weights are a non-linear function of various parameters In this section, we shed more light on the sensitivity of the weights to those parameters. We first compute the partial derivatives of the weight of the high–risk asset A with respect to each of the input parameters.[5] These derivatives show the direction and the magnitude of the effect of an infinitesimal change in the input parameters. We then use a numerical evaluation to quantify the effects of larger changes in the magnitude of the parameters.

3.1. *High–risk asset weight partial sensitivity to the input parameters*

A crucial feature of the momentum interpretation of the MBo2 weight allocation is that the weight of the high–risk asset is an increasing function of the relative price of the high–risk asset regarding the price of the low–risk asset. It implies that the partial derivative of the high–risk asset weight with respect to its relative price must be positive. We prove this in Appendix D, where we obtain the following expression:

[5] The derivatives for the low–risk asset B follow directly from $w_{B,t} \equiv 1 - w_{A,t}$.

$$\frac{\partial w_{A,t}}{\partial P_{A/B,t|t_0}} = \underbrace{\frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}}_{>0}$$

$$\times \underbrace{\left\{(1-\Phi(d_2))\Phi(d_1) + \frac{1}{\sigma_{A-B}\sqrt{\tau}}\left[(1-\Phi(d_2))\phi\left(d_1\right) + \phi\left(d_2\right)\right]\right\}}_{\geq 0}. \tag{4}$$

In Appendices E–H, we show that the partial derivatives of the weight of the high–risk asset with respect to the other parameters (return's volatility, correlation, and the time to maturity of the Margrabe option) have a common structure, namely:

$$\frac{\partial w_{A,t}}{\partial \sigma_A} = \kappa \times \underbrace{\left(\sigma_A - \sigma_B\rho_{A,B}\right)}_{>0} \tag{5}$$

$$\frac{\partial w_{A,t}}{\partial \sigma_B} = \kappa \times \left(\sigma_B - \sigma_A\rho_{A,B}\right) \tag{6}$$

$$\frac{\partial w_{A,t}}{\partial \rho_{A,B}} = (-1) \times \kappa \times \sigma_A\sigma_B \tag{7}$$

$$\frac{\partial w_{A,t}}{\partial \tau} = \kappa \times \frac{1}{2}\sigma_{A-B}\,, \tag{8}$$

where we use that, by definition of the high–risk asset, $\sigma_A > \sigma_B$ and thus $\sigma_A - \sigma_B\rho_{A,B} > 0$. The common parameter κ is given by:

$$\kappa \equiv \underbrace{\frac{P_{A/B,t|t_0}\sqrt{\tau}}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}}_{>0}$$

$$\times \left[(1-\Phi(d_2))\phi(d_1)\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right) - \Phi(d_1)\phi(d_2)\left(\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau} + 1\right)\right]. \tag{9}$$

A sufficient condition for κ to be negative is that $\ln(P_{A/B,t|t_0}) \geq \sigma^2_{A-B}\tau$, which means that $P_{A/B,t|t_0} \geq \exp(\sigma^2_{A-B}\tau)$. When the high–risk asset outperforms the low–risk asset, we thus have that an increase of the volatility of the high–risk leads to a decrease of the high–risk asset's weight in the MBo2 replicating portfolio. Such directional impact is similar to those of the time to the maturity and the volatility of the low–risk asset (under the condition that $\rho_{A,B} < \sigma_B/\sigma_A$). Such impact is opposite to those of the correlation of two assets' returns. Regarding the magnitude of the impact, the volatility of the high–risk asset's return and the time to maturity have a big impact on the weight of the high–risk asset in the MBo2 replicating portfolio. Such impact is bigger than those of the volatility of the low–risk asset's return and the correlation of two assets' returns.

3.2. Numerical study of the sensitivity of $w_{A,t}$ to the input parameters

We now illustrate the sensitivity of the MBo2 weights to larger (non–infinitesimal) changes in the parameters in a stylized setup. We first describe the calibration of the parameters and then discuss the results.

Setup. As the reference scenario, we assume that the relative price of asset A over B is 1.1 ($P_{A/B,t|t_0} = 1.1$). The volatility of the high–risk and low–risk asset return series is 25% and 6%, respectively. The correlation is -0.2. The initial investment horizon of the MBo2 investment strategy is 12 months and the evaluation period is the fifth one (seven periods until the exercise of the option). In terms of the key parameters of (5)–(8), we have that, $\kappa = -0.16$ and $\sigma_B - \rho_{A,B}\sigma_A = 0.11$. Table 2 reports the results of the numerical study of the joint impact of the input parameters on the weight of the high–risk asset in the MBo2 replicating portfolio.

Sensitivity to the relative performance. Let us first investigate how the relative performance of the high–risk asset compared to the low–risk asset affects the high–risk asset weight, while keeping all other variables constant. Since the partial derivative of the weight of the high–risk asset with respect to the relative price is positive (as (4)), the weight on the high–risk asset increases when the relative price increases, *ceteris paribus*. This can be seen in the column of the numbers in bold in Panel A of Table 2. The weight varies from 0.51% to 95.59% for a range of relative prices from 0.6 to 1.4, respectively. For all columns, we see that the weight of the high–risk asset increases when the relative price increases.

Sensitivity to the volatility. The second block of Panel A shows that the weight is also very sensitive to the volatility of the high–risk asset's return.[6] At a relative price of 1.1, the weights of the high–risk asset are 93.05%, 68.68%, and 61.44% for the volatility of the high–risk asset's return at 5%, 25%, and 45%, respectively. The decrease in weight of the high–risk asset when its volatility increases reflects the decrease in probability that the relative price at maturity still exceeds one.

Comparing the bold column (the volatility of the high–risk asset's return is at 25%), and changing the volatility of the low–risk asset from 4% (the middle column in the first block) to 8% (the middle column in the third block), the weight does not change much (*e.g.*, at the relative price level of 1.2, the weights are 83.01% and 81.62% if the volatility of the low–risk asset is at 4% and 8%, respectively versus 82.35% of the base case where the volatility of the low–risk asset is 6%.).

Sensitivity to the correlation. From (7), the effect of the correlation is opposite to the sign of κ. As κ in our reference case is negative, an increase in correlation leads to an increase in the weight. This effect is however weak. In fact, as can be seen in Panel B of Table 2, the correlation has less impact on the weight of the high–risk asset in the MBo2 replicating portfolio. Particularly, when the relative price is 1.1, *ceteris paribus*, the weight of the high–risk asset is 67.18%, 68.68%, and 70.68% for correlation values of -0.7, -0.2, and 0.3, respectively.

Sensitivity to the time to maturity. According to (8), the directional impact of the high–risk asset weight positively relates to κ. In our reference case, κ is negative. When the time to maturity decreases, the high–risk asset weight thus increases. This can be seen in Panel C where the relative price is 1.1, *ceteris paribus*, the high–risk asset weight increases slightly (65.28%, 68.68%, and 76.76%) for decreasing values of time to maturity (eleven, seven, and three periods until the maturity). The impact is thus moderate.

[6]We do not report the second block in Panel B and Panel C as they are the same as in Panel A.

Table 2: Numerical study of the sensitivity of the high–risk asset weight with respect to parameters
This table presents the weight of the high–risk asset in the MBo2 replicating portfolio using the MBo2 strategy for various values of the input parameters. Panel A shows the sensitivity of the weight of the high–risk asset with respect to the volatility of the low–risk asset's return. The sensitivity with respect to the correlation of two assets' returns is presented in Panel B. Panel C shows the sensitivity with respect to the time to maturity of the option. The base case of our analysis is presented in the middle column and marked in bold. Middle columns of Panel B and Panel C are skipped as they are the same as those of Panel A. See Section 3 for details.

Panel A: $w_{A,t}$ for various values of the low–risk asset's return volatility σ_B

	$\sigma_B = 4\%$ and $\sigma_A =$					$\sigma_B =$ **6%** and $\sigma_A =$					$\sigma_B = 8\%$ and $\sigma_A =$				
$P_{A/B,t\|t_0}$	5%	15%	25%	35%	45%	5%	15%	**25%**	35%	45%	5%	15%	25%	35%	45%
0.60	0.00	0.00	0.41	2.56	5.97	0.00	0.00	**0.51**	2.76	6.19	0.00	0.01	0.63	3.00	6.44
0.70	0.00	0.18	3.18	8.53	13.70	0.00	0.29	**3.55**	8.90	13.99	0.00	0.47	4.00	9.31	14.31
0.80	0.00	3.32	12.17	19.46	24.60	0.03	4.12	**12.81**	19.86	24.86	0.20	5.11	13.54	20.31	25.14
0.90	2.33	19.20	29.02	34.17	37.25	5.11	20.51	**29.53**	34.43	37.40	8.58	21.92	30.09	34.71	37.56
1.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	**50.00**	50.00	50.00	50.00	50.00	50.00	50.00	50.00
1.10	96.41	78.45	69.15	64.39	61.57	93.05	77.19	**68.68**	64.16	61.44	89.20	75.84	68.15	63.90	61.29
1.20	99.97	93.35	83.01	75.94	71.29	99.76	92.24	**82.35**	75.57	71.06	99.08	90.94	81.62	75.16	70.81
1.30	100.00	98.44	91.46	84.40	79.02	100.00	97.93	**90.87**	83.99	78.74	99.96	97.25	90.20	83.54	78.43
1.40	100.00	99.71	96.01	90.21	84.91	100.00	99.54	**95.59**	89.83	84.62	100.00	99.29	95.09	89.40	84.30

Panel B: $w_{A,t}$ for various values of the return correlation $\rho_{A,B}$

	$\rho_{A,B} = -0.7$ and $\sigma_A =$					$\rho_{A,B} =$ **-0.2** and $\sigma_A =$					$\rho_{A,B} = 0.3$ and $\sigma_A =$				
$P_{A/B,t\|t_0}$	5%	15%	25%	35%	45%	5%	15%	**25%**	35%	45%	5%	15%	25%	35%	45%
0.60	0.00	0.03	0.94	3.65	7.23						0.00	0.00	0.20	1.90	5.10
0.70	0.00	0.75	4.94	10.39	15.28						0.00	0.05	2.18	7.24	12.54
0.80	0.18	6.32	14.96	21.43	25.99						0.00	1.93	10.23	17.97	23.53
0.90	8.38	23.43	31.14	35.40	38.04						1.69	16.32	27.38	33.21	36.63
1.00	50.00	50.00	50.00	50.00	50.00						50.00	50.00	50.00	50.00	50.00
1.10	89.42	74.40	67.18	63.27	60.85						97.26	81.27	70.68	65.27	62.13
1.20	99.14	89.45	80.22	74.13	70.07						99.99	95.48	85.04	77.33	72.25
1.30	99.97	96.37	88.86	82.37	77.51						100.00	99.24	93.18	85.91	80.17
1.40	100.00	98.91	94.05	88.28	83.33						100.00	99.91	97.16	91.57	86.09

Panel C: $w_{A,t}$ for various values of the time to maturity of the option τ

	$\tau = 3$ and $\sigma_A =$					$\tau =$ **7** and $\sigma_A =$					$\tau = 11$ and $\sigma_A =$				
$P_{A/B,t\|t_0}$	5%	15%	25%	35%	45%	5%	15%	**25%**	35%	45%	5%	15%	25%	35%	45%
0.60	0.00	0.00	0.01	0.21	1.13						0.00	0.08	1.89	5.97	10.46
0.70	0.00	0.00	0.34	2.23	5.47						0.00	1.32	7.22	13.70	18.90
0.80	0.00	0.44	4.42	10.33	15.70						0.29	8.12	17.95	24.60	29.00
0.90	0.65	10.67	20.96	27.47	31.67						9.52	25.39	33.20	37.25	39.67
1.00	50.00	50.00	50.00	50.00	50.00						50.00	50.00	50.00	50.00	50.00
1.10	98.77	86.99	76.76	70.59	66.69						88.20	72.55	65.28	61.57	59.36
1.20	100.00	98.41	91.84	84.93	79.49						98.81	87.36	77.35	71.29	67.46
1.30	100.00	99.90	97.73	93.09	88.14						99.94	94.95	85.93	79.01	74.22
1.40	100.00	100.00	99.47	97.10	93.47						100.00	98.20	91.59	84.91	79.73

Visualization of sensitivity to parameters. To gain a better intuition on the sensitivity of the weight to the input parameters, we plot in Figures 2 and 3 the high-risk asset weight as a function of the relative price, return volatilities, correlation, and time to maturity, respectively. In Figure 2 we see that when the relative price is at 1.0, the weight of the high–risk asset is always 50% regardless of the value of the other parameters. In case of a relative price of 0.9, an increase in volatilities of two assets' returns and the number of periods until maturity lead to an increase of the weight of the high–risk asset, while an increase of the correlation leads to a slight decrease of the weight. The opposite holds for a relative price of 1.1.

Figure 2: Sensitivity of the high–risk asset weight with respect to parameters

These plots display the weight of the high–risk asset in the MBo2 replicating portfolio ($w_{A,t}$) for various levels of the input parameters. Each plot considers three different values of the relative prices (0.9, 1.0 and 1.1) and different values of high–risk asset volatility (top left), correlation (top right), the low–risk asset volatility (bottom left), and time to maturity (bottom right). See Table 2 for details.

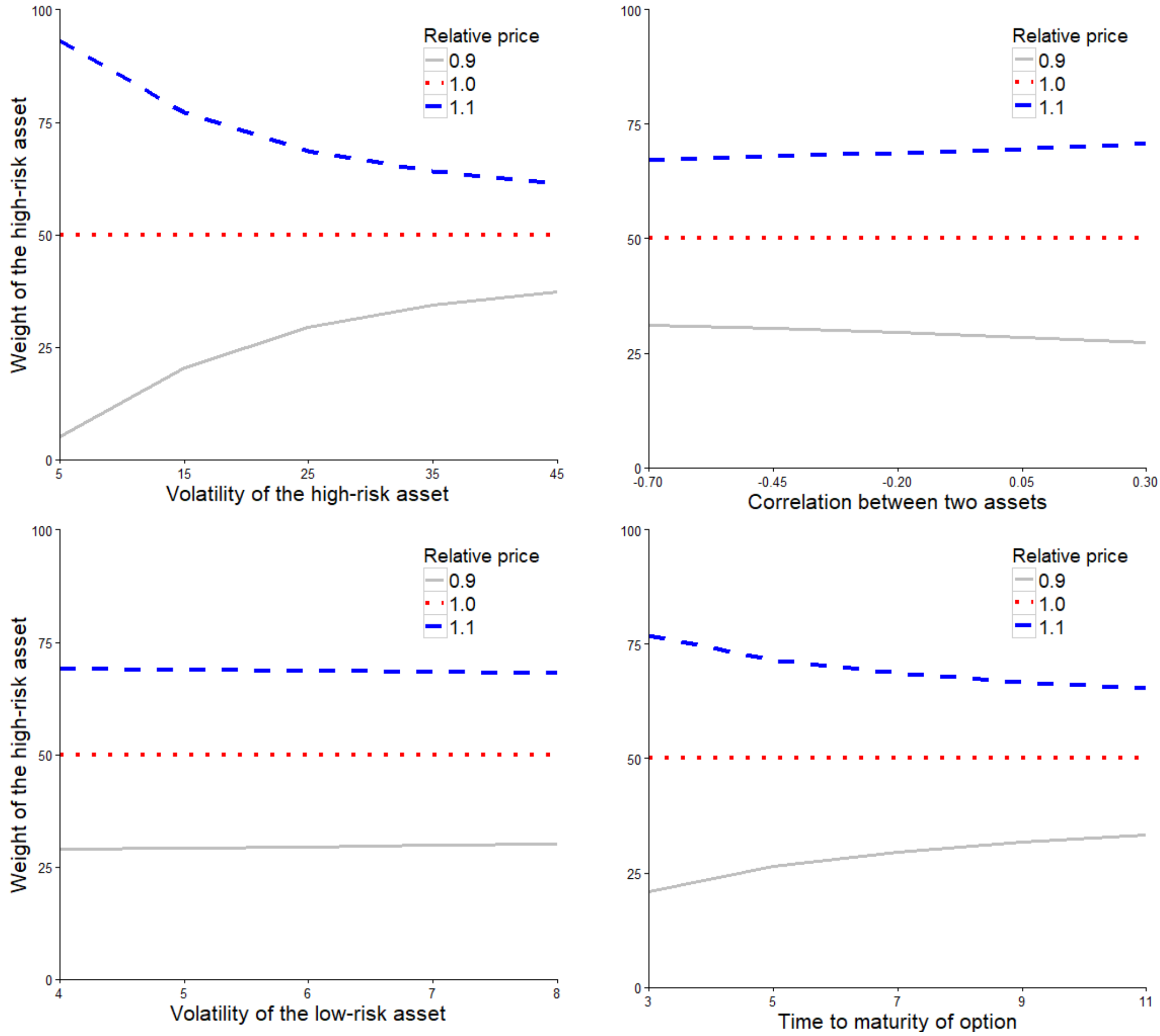


Figure 3: Sensitivity of the high–risk asset weight with respect to the relative price
This plot shows the weight of the high–risk asset ($w_{A,t}$) with respect to the relative price and two extreme Margrabe option parameter values. Each plot considers the weight of the high–risk asset sensitivity to a range of relative prices (from 0.6-1.4) and two extreme (high and low) values of high–risk asset's return volatility (top left), correlation (top right), low–risk asset's return volatility (bottom left), and time to maturity of the option (bottom right). See Table 2 for details.

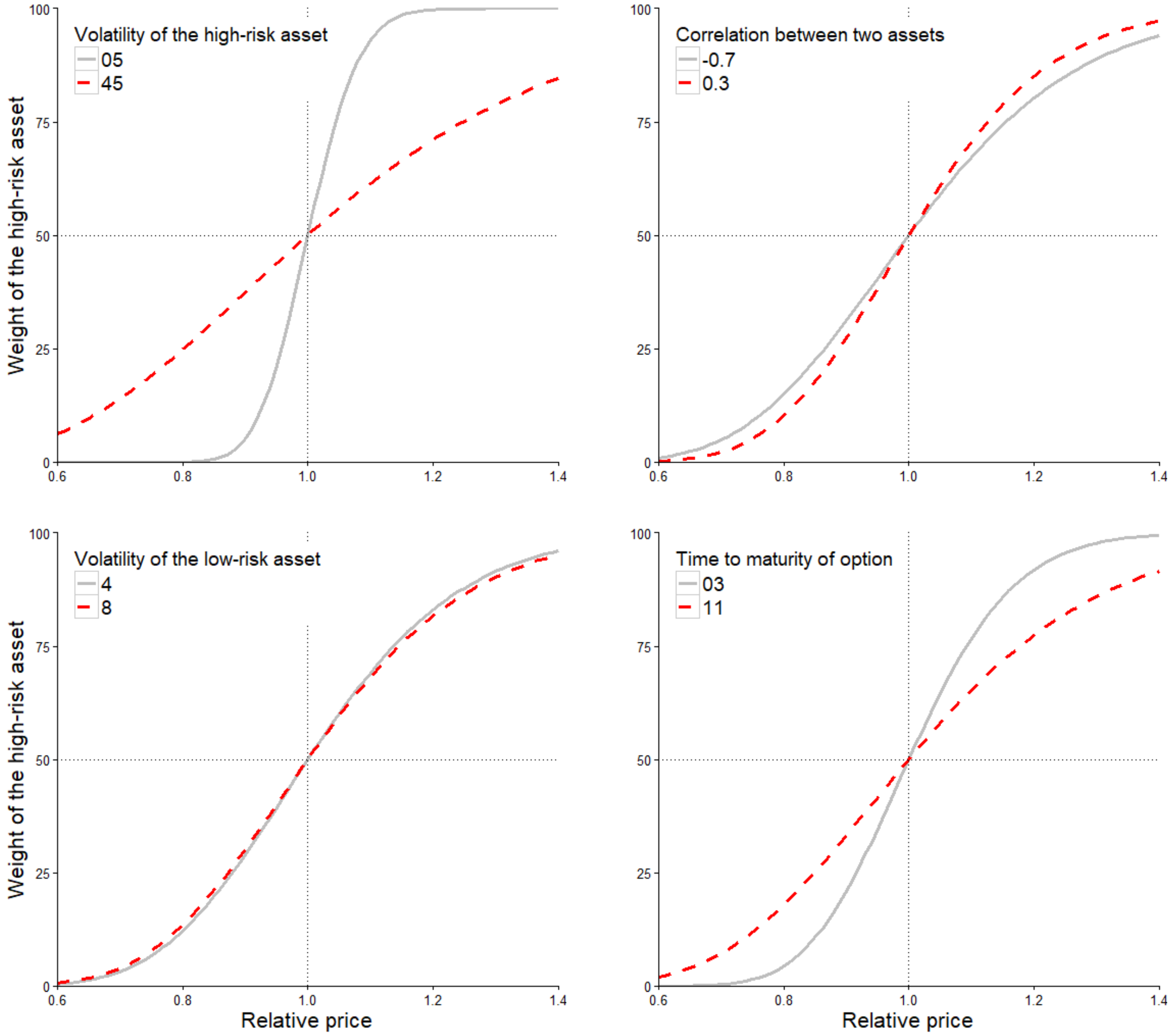


Table 2 illustrates the dependence of the high–risk asset weight on input parameters for a small grid of values. In Figure 3, we plot the high–risk asset weight as a continuous function of the relative price for various values of the other parameters. Figure 3 shows that the weight of the high–risk asset is extremely different in cases of low and high value of the high–risk's asset volatility (5% and 45%, respectively). In particular, when the volatility of the high–risk asset's return is 5%, the high–risk asset weight increases from 5.11% to 50% when the relative price increases from 0.9 to 1.0. Meanwhile, the high–risk asset weight only increases from and 37.40% to 50% when the volatility of the high–risk asset's return is 45%.

Overall, at the high volatility of the high–risk asset's return (45%), the high–risk asset weight in the replicating portfolio is almost a linear function of the relative price with a positive slope. This is not the case for the low volatility level of the low–risk asset, where the weight of the high–risk asset does not change.

Regarding the correlation between two assets' returns, one can note that the gap between the two lines of the weight of the high–risk asset is smaller than those of the volatility of the high–risk asset's returns and the time to maturity of the option. The impact on the weight of the high–risk asset is, therefore, less noticeable than those of the volatility and the time to maturity of the option.

The difference between the two extreme values of the time to maturity of the Margrabe option is less obvious than the volatility of the high–risk asset but clearer than the correlation. For all parameters, we see the non–linear and asymmetric relation between the relative price and the weight of the high–risk asset in the MBo2 replicating portfolio. In addition, when the relative price is either extremely high/low or close to one, the differences are smaller than those in the other cases.

4. The non–linearity between the performance of the high–risk asset and the MBo2 strategy

In this section, we study the portfolio performance of the MBo2 strategy for a one–year and a five–year investment horizon. We illustrate that the MBo2 is an investment strategy that benefits from the upside potential of the high–risk asset and the downside protection of the low–risk asset.

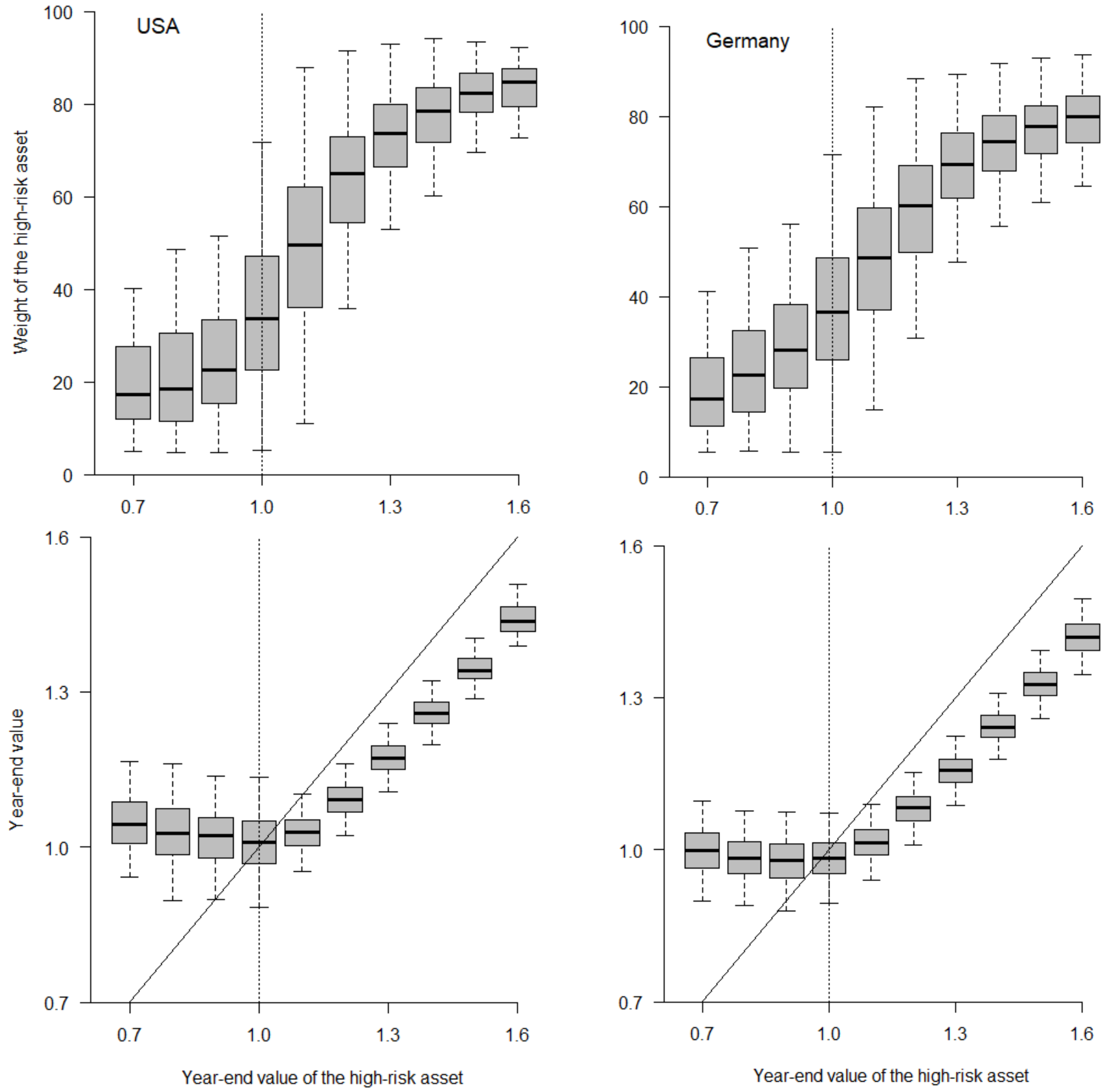
4.1. Simulation methodology

Investors in the MBo2 solutions typically have a medium to long–term investment horizon. Typical investment horizons are one year or longer. The drawback of such a long investment horizon is that the historical data provide only a small number of evaluation samples. To overcome this problem, we follow Perold and Sharpe (1995) and Ardia et al. (2016) and backtest the MBo2 strategy by simulating M artificially generated price–paths of the high–risk asset for two investment horizons: one year and five years. Based on Jegadeesh and Titman (1993), the momentum strategy requires sampling windows of at least three months to preserve the positive autocorrelation in the return series. We therefore apply a block–bootstrap with block length of four and six months (for one–year and five–year investment horizons, respectively) which are sufficient to strike a balance between preserving the momentum in the returns, and allowing for heterogeneity in the simulated paths. Following Perold and Sharpe (1995) and Ardia et al. (2016), we simulate $M = 10{,}000$ price–paths.

4.2. Results

Figure 4 displays the results for the one–year investment horizon. We show the year–end value of the buy–and–hold strategy on the high–risk asset (the horizontal axis) and the weight on the high–risk asset of the MBo2 strategy (the top charts) as well as the portfolio performance (the bottom charts). Results are shown for the investment in the US and German markets.

Figure 4: Portfolio composition and performance of the MBo2 strategy for a one–year investment horizon
These plots show the average high–risk asset weights and the corresponding cumulative values over one–year investment of the MBo2 strategy. The reference performance is the buy–and–hold strategy on the high–risk asset (the full line). These strategies invest in the US market (top left and bottom left charts) and the German market (top right and bottom right charts). Data is simulated by using the block–bootstrap technique. See Section 4 for details.



Consider first the dependence of the high–risk asset weight on the year–end value of the high–risk asset in the two top charts. We see increasing values of the high–risk asset weight for increasing year-end values of the buy–and–hold strategy on the high–risk asset. Note that when the high–risk asset ends below one, the MBo2 strategy invests less than 50% in the high–risk asset on average (and more than 50% in the low–risk asset). It therefore indicates a downside protection in the bearish market. The weights of the high–risk asset increase to about 85% when the high–risk asset ends at 1.6 in the bullish market. It implies a moderate uptrend potential of the MBo2 strategy.

Regarding the portfolio performance (two charts at the bottom), the full line represents the year–end value of the buy–and–hold strategy on the high–risk asset. As we can see, the cumulative value is a linear function of the year–end value of the high–risk asset (the slope of the line is one). Different from the buy–and–hold strategy, the year–end value of the MBo2 strategy has a non–linear dependence on the year–end value of the high–risk asset. It implies a dependence on both of the high– and low–risk assets' price. At the extreme low year–end value of the high–risk asset (*e.g.* 0.7, or 30% of drawdown, equivalently) the weight on the high–risk asset is less than 50%. The portfolio value is then close to 1.05 (5% gain). The portfolio value gradually decreases when the year–end value of the high–risk asset increases up to one. When the year–end value of the high–risk asset is greater than one, the year–end value of the MBo2 strategy is slightly lower than those of the high–risk asset.

In Figure 5, we present results of the five–year investment horizon. We see similar patterns of the weight chart and the portfolio performance compared with the performance of the one–year investment strategy. The average weights on the high–risk asset increase in stable paths in corresponding with increasing final–value of the buy–and–hold strategy on the high–risk asset. Regarding the portfolio performance, when the buy–and–hold strategy on the high–risk asset yields a minimum cumulative value of 0.4 (60% of drawdown), the MBo2 strategy yields a cumulative value of 1.2 (20% gain) on average. For the other cumulative values from 1.6 to 3.1, the buy–and–hold strategy on the high–risk asset yields slightly better cumulative values than the MBo2 strategy. In the best case, the buy–and–hold strategy on the high–risk asset has a maximum cumulative value of 3.4 (240% gain) while the cumulative value of the MBo2 strategy is 2.4 (140% gain) on average. Similar to the one–year MBo2 investment, the five–year MBo2 strategy combines the upside potential and the downside protection. It outperforms the buy–and–hold strategy on the high–risk asset during the downtrend market and underperforms during the uptrend market.

5. Improving the design of the MBo2 strategy

5.1. Methodology

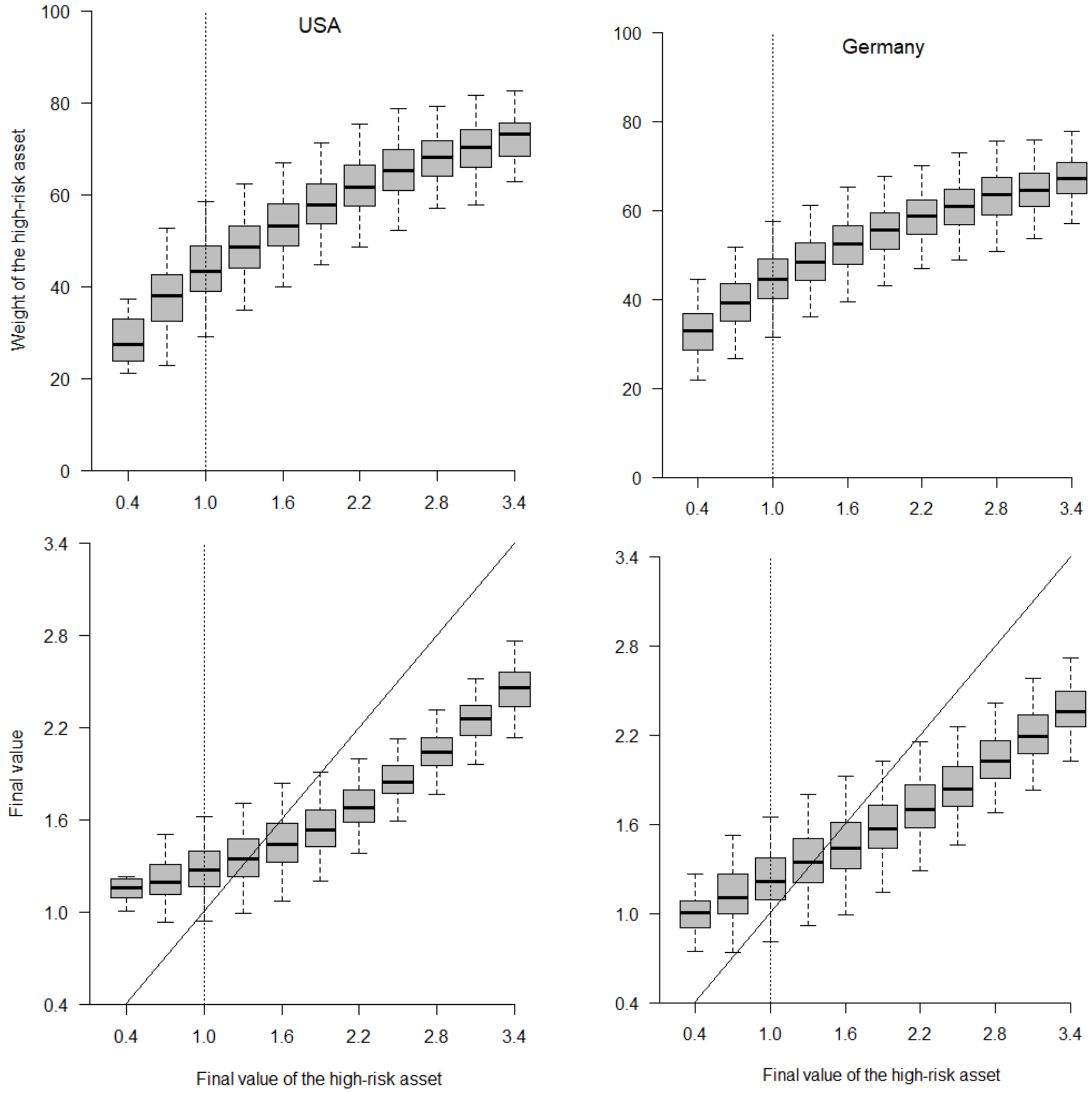
The current implementation of the MBo2 strategy uses *backward–looking* estimations of volatilities, correlation, and a yearly rebalancing date in January. In this section, we investigate whether we can improve the MBo2 performance by considering alternative implementations.

First, we apply the *forward–looking* approaches of volatilities and correlation to relax the assumption of using *backward–looking* estimators. In the first approach (Alt#1), the implied volatility data of the S&P 500 and DAX index (VIX and VDAX) from CBOE is used instead of the rolling volatility of the high–risk asset's returns. In the second approach (Alt#2), volatilities of two assets' returns are forecasted using the GARCH(1,1) model of Bollerslev (1986), while the correlation is estimated in the dynamic conditional correlation (DCC) model of Engle (2002). The volatilities are forecasted for T different horizons (from 1 to T). The average of these values is used in the optimization over the period $[t_{0,k}, t_{0,k+1}]$ (see the iterated variance approach of Ghysels et al. (2009) for details).

Second, we relax the assumption on the rebalancing strategy. In Alt#3, we consider a longer investment horizon in the option (T = 24 months). In practice, traders also use a one–month investment horizon for the option (T = 1 month). Therefore, we apply an alternative approach using a one–month investment horizon of the option where volatilities and correlation are also updated every month (Alt#4). The formula that links the time–varying weights and correlation parameter is the same as in (3), but we replace the constant parameters with their time–varying alternatives.

Regarding the base date, we set it in January as this is standard practice (Ariel (1990) and Thaler (1987)). In the financial market, Bouman and Jacobsen (2002) analyze the long–lasting axiom: 'Sell in May, and

Figure 5: Portfolio composition and performance of MBo2 strategy for a five–year investment horizon
These plots show the average high–risk asset weights and the corresponding cumulative values over five–year investment of the MBo2 strategy. The reference performance is the buy–and–hold strategy on the high–risk asset (the full line). These strategies invest in the US market (top left and bottom left charts) and the German market (top right and bottom right charts). Data is simulated by using the block–bootstrap technique. See Section 4 for details.



go away'. They find that the stock returns during the period May– October is significantly lower than the remainder period (September– April) for 36 out–of the 37 studied countries, including the United States and Germany. We, therefore, test an alternative implementation which rebalances in May (Alt#5). In Alt#6, we implement a strategy to rebalance the portfolio on a daily basis. The specifications of the alternative implementations considered of the MBo2 strategy are summarized in Panel A of Table 3.

Table 3: Alternative implementations for the MBo2 strategy
This table presents the traditional approach of the MBo2 strategy and six alternative implementations to measure the forward–looking volatilities, correlation of the high–risk and low–risk assets' returns, length of the option, reset month, and rebalance strategy (Panel A). Rolling approach is estimated from rolling windows of returns series (volatilities are annualized). In the first approach (Alt#1), the implied volatility of the high–risk asset's return (the implied volatility index of S&P 500 index- VIX provided by CBOE- and DAX- VDAX) is applied. In Alt#2, the volatilities and correlation are forecasted using the GARCH(1,1) and DCC models, respectively. Alt#3 and Alt#4 modify the investment horizon of the option (T = 24 months and 1 month, respectively). Alt#5 assumes that the reset month is in May. In Alt#6, we rebalance the weight on a daily basis and aggregate the daily returns to make performance criteria comparable with other implementations. The performance of the strategies is shown in Panel B (the US market) and Panel C (the German market). See Table 1 for details.

Panel A: Implementations							
Approach	σ_A	σ_B	$\rho_{A,B}$	T	Reset	Rebalance	
MBo2	Rolling	Rolling	Rolling	12	Jan	Monthly	
Alt#1	Implied	Rolling	Rolling	12	Jan	Monthly	
Alt#2	GARCH	GARCH	DCC	12	Jan	Monthly	
Alt#3	Rolling	Rolling	Rolling	24	Jan	Monthly	
Alt#4	Rolling	Rolling	Rolling	1	Jan	Monthly	
Alt#5	Rolling	Rolling	Rolling	12	May	Monthly	
Alt#6	Rolling	Rolling	Rolling	12	Jan	Daily	
Panel B: Portfolio performance on the US market							
Approach	CVal	Mean	Std	SR	MaxDD	MVaR	TO
MBo2	9.51	10.69	8.52	0.77	14.30	3.35	62.53
Alt#1	9.05	10.45	8.05	0.78	11.08	3.06	52.49
Alt#2	9.08	10.46	8.46	0.75	13.71	3.34	62.50
Alt#3	8.29	10.01	8.69	0.68	13.55	3.42	50.24
Alt#4	10.08	10.99	9.14	0.75	15.36	3.71	84.54
Alt#5	9.25	10.56	7.78	0.82	8.91	2.63	57.50
Alt#6	7.43	9.47	8.30	0.64	11.60	3.16	75.42
Panel C: Portfolio performance on the German market							
Approach	CVal	Mean	Std	SR	MaxDD	MVaR	TO
MBo2	9.48	10.68	12.00	0.59	20.53	4.49	57.34
Alt#1	9.77	10.83	11.77	0.61	19.08	4.38	54.69
Alt#2	9.97	10.93	12.04	0.61	20.43	4.51	56.81
Alt#3	8.87	10.35	11.75	0.57	19.79	4.28	48.09
Alt#4	10.25	11.07	12.73	0.58	22.25	4.87	78.48
Alt#5	9.23	10.54	10.91	0.64	17.40	4.12	60.92
Alt#6	9.20	10.57	12.15	0.57	22.43	4.36	66.06

5.2. Performance

In Table 3, we present the performance of alternative implementations of the United States (Panel B) and German markets (Panel C). For the United States market, four implementations (Alt#1, 2, 5, and 6) yield lower volatilities and drawdowns than the traditional MBo2 strategy. Alt#5, which relaxes the assumption of rebalancing monthly in May, yields a slightly higher Sharpe ratio than the traditional MBo2 strategy (0.82). Regarding the portfolio return, Alt#4 (using the investment horizon on the option is one months instead of 12 months as the traditional and other implementations) yields the highest cumulative end–value and annualized return ($10.08 and 10.99%). Its volatility and drawdown are therefore slightly higher than other implementations (9.14% and 15.36%).

For the investment in the German market, we find almost the same conclusions, where Alt#1, 3 and 5 reduce the annualized volatility and drawdown compared to the base MBo2. Meanwhile, Alt#4 yields the highest return (11.07% compared to 10.68% of the base MBo2).

In summary, we recommend to consider an alternative implementation of the MBo2 strategy by the use of option–implied volatility (Alt#1) instead of the historical volatility. It yields a higher risk–adjusted return, lower volatility, lower drawdown and slightly lower turnover than the traditional implementation.

6. Conclusion

In the bond–equity investment problem, the MBo2 strategy is a tactical asset allocation strategy. The rule behind the strategy is to dynamically allocate the capital on the bond and equity given by the information on their relative price, volatilities, correlations of their returns, and the time to maturity of the exchange option. While it is somewhat popular in practice, its properties have not been studied in detail. In this paper, we provide practitioners with better insights on the MBo2 strategy.

First, we derive explicit formulae of the impact of the inputs of the strategy on the portfolio composition and the portfolio performance. We show that among the parameters of the strategy, the relative price and the volatility of the high–risk asset's return are the most influential parameters driving the portfolio weight.

Second, we use the simulated data using the block–bootstrap technique to study the sensitivity of the portfolio weights and performance of the MBo2 strategy. It confirms the upside potential and the drawdown protection over the one– and five–year investment horizons.

Third, we investigate various implementations relaxing implementations of the traditional strategy. The analysis shows that the implementation using the option–implied volatility yields a similar return, lower volatility, higher Sharpe ratio, lower drawdown, and lower turnover than the traditional implementation.

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Appendix A. Equivalent expression for the value of the MBo2 replicating portfolio

The MBo2 strategy consists of investing in an underlying asset and buying the Margrabe option to exchange an asset for another asset. Using the valuation formula (1), we can construct a replicating portfolio by investing directly in asset A and asset B. The value of the MBo2 replicating portfolio is equal to the sum of the price of asset B and the option price to exchange asset B for asset A (see Rubinstein and Leland (1981)):

$$P_{\text{MBo2},t} \equiv P_{B,t|t_0} + C_t^{B\to A}\,. \tag{A.1}$$

To show that $P_{\text{MBo2},t} = P_{A,t|t_0} + C_t^{A\to B}$, we first rewrite:

$$P_{B,t|t_0} + C_t^{B\to A} = P_{B,t|t_0} + P_{A,t|t_0}\Phi\left(\frac{\ln(P_{A/B,t|t_0}) + \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right) - P_{B,t|t_0}\Phi\left(\frac{\ln(P_{A/B,t|t_0}) - \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right)\,. \tag{A.2}$$

Using $\ln(x) = -\ln(1/x)$ with $x > 0$ and $\Phi(x) = 1 - \Phi(-x)$, we can write (A.2) as:

$$\begin{aligned}
P_{B,t|t_0} + C_t^{B\to A} &= P_{A,t|t_0}\Phi\left(\frac{-\ln(P_{B/A,t|t_0}) + \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right) + P_{B,t|t_0}\left(1 - \Phi\left(\frac{-\ln(P_{B/A,t|t_0}) - \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right)\right) \\
&= P_{A,t|t_0}\left(1 - \Phi\left(\frac{\ln(P_{B/A,t|t_0}) - \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right)\right) + P_{B,t|t_0}\Phi\left(\frac{\ln(P_{B/A,t|t_0}) + \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right) \\
&= P_{A,t|t_0} + P_{B,t|t_0}\Phi\left(\frac{\ln(P_{B/A,t|t_0}) + \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right) - P_{A,t|t_0}\Phi\left(\frac{\ln(P_{B/A,t|t_0}) - \frac{1}{2}\sigma^2_{A-B}\tau}{\sigma_{A-B}\sqrt{\tau}}\right) \\
&= P_{A,t|t_0} + C_t^{A\to B}\,.
\end{aligned} \tag{A.3}$$

The value of the MBo2 replicating portfolio is:

$$P_{\text{MBo2},t} = P_{A,t|t_0}\Phi(d_1) + P_{B,t|t_0}(1 - \Phi(d_2))\,. \tag{A.4}$$

Appendix B. Composition of the MBo2 replicating portfolio

Given the value of the Margrabe replicating portfolio in Appendix A, the question is how many units of asset A (n_A) and asset B (n_B) do we need to buy to have a portfolio with the value equal to the value of the MBo2 replicating portfolio? From equal–value conditions, we have:

$$P_{A,t}n_{A,t} + P_{B,t}n_{B,t} = P_{\text{MBo2},t} = P_{A,t|t_0}\Phi(d_1) + P_{B,t|t_0}(1 - \Phi(d_2))\,. \tag{B.1}$$

We have that $P_{A,t} = P_{A,t|t_0}P_{A,t_0}$ and $P_{B,t} = P_{B,t|t_0}P_{B,t_0}$, which implies:

$$P_{A,t_0}n_{A,t} = \Phi(d_1) \tag{B.2}$$

$$P_{B,t_0}n_{B,t} = 1 - \Phi(d_2)\,. \tag{B.3}$$

The weight of asset A in the MBo2 replicating portfolio is:

$$w_{A,t} \equiv \frac{P_{A,t}n_{A,t}}{P_{A,t}n_{A,t} + P_{B,t}n_{B,t}} = \frac{P_{A,t|t_0}\Phi(d_1)}{P_{A,t|t_0}\Phi(d_1) + P_{B,t|t_0}(1 - \Phi(d_2))} = \frac{P_{A/B,t|t_0}\Phi(d_1)}{P_{A/B,t|t_0}\Phi(d_1) + (1 - \Phi(d_2))}\,, \tag{B.4}$$

and the weight of asset B in the MBo2 replicating portfolio is:

$$w_{B,t} \equiv \frac{P_{B,t}n_{B,t}}{P_{A,t}n_{A,t} + P_{B,t}n_{B,t}} = \frac{P_{B,t|t_0}(1 - \Phi(d_2))}{P_{A,t|t_0}\Phi(d_1) + P_{B,t|t_0}(1 - \Phi(d_2))} = \frac{(1 - \Phi(d_2))}{P_{A/B,t|t_0}\Phi(d_1) + (1 - \Phi(d_2))}\,. \tag{B.5}$$

Note that $w_{B,t} = 1 - w_{A,t}$.

To prove (3), we further recall that under the Black–Scholes assumptions, we have that, at time t, the standardized variable:

$$z \equiv \frac{\ln\left(\frac{P_{A/B,T|t}}{P_{A/B,t|t_0}}\right) + \frac{\sigma_{A-B}^2}{2}\tau}{\sigma_{A-B}\sqrt{\tau}} \sim N(0,1)\,. \tag{B.6}$$

At maturity, the expected value of the relative price conditional on the relative price is greater than one is:

$$
\begin{aligned}
\mathbb{E}[P_{A/B,T|t}|P_{A/B,T|t}>1] &= \mathbb{E}[P_{A/B,T|t}|\ln(P_{A/B,T|t})>0] \\
&= \int_{\ln(P_{A/B,T|t})=0}^{+\infty} P_{A/B,T|t} f(\ln(P_{A/B,T|t}))d(\ln(P_{A/B,T|t})) \\
&= P_{A/B,t|t_0}\frac{1}{\sqrt{2\pi}}\int_{z=-d_2}^{+\infty} e^{-\frac{z^2-2z\sigma_{A-B}\sqrt{\tau}+\sigma_{A-B}^2\tau}{2}}dz \\
&= P_{A/B,t|t_0}\frac{1}{\sqrt{2\pi}}\int_{-d_2}^{+\infty} e^{-\frac{(z-\sigma_{A-B}\sqrt{\tau})^2}{2}}dz \\
&= P_{A/B,t|t_0}\frac{1}{\sqrt{2\pi}}\int_{y=-d_2-\sigma_{A-B}\sqrt{\tau}}^{+\infty} e^{-\frac{y^2}{2}}dy \\
&= P_{A/B,t|t_0}\frac{1}{\sqrt{2\pi}}\int_{-d_1}^{+\infty} e^{-\frac{y^2}{2}}dy \\
&= P_{A/B,t|t_0}(1-\Phi(-d_1)) = P_{A/B,t|t_0}\Phi(d_1)\;, \qquad \text{(B.7)}
\end{aligned}
$$

where $f(x)$ is the probability density function of the normal random variable $\ln(P_{A/B,T|t})$ with mean $\ln(P_{A/B,t|t_0}) - \frac{\sigma_{A-B}^2}{2}\tau$ and variance $\sigma_{A-B}^2\tau$. Note that we use $y = z - \sigma_{A-B}\sqrt{\tau}$ and $dy = dz$ in the fifth equation.

The probability that the relative price will be higher than one at maturity is:

$$
\begin{aligned}
\mathbb{P}[P_{A/B,T|t}>1] = \mathbb{P}[\ln(P_{A/B,T|t})>0] &= \int_{\ln(P_{A/B,T|t})=0}^{+\infty} f(\ln(P_{A/B,T|t}))d(\ln(P_{A/B,T|t})) \\
&= \int_{z=-d_2}^{+\infty} f(z)dz = 1-\Phi(-d_2) = \Phi(d_2)\,. \qquad \text{(B.8)}
\end{aligned}
$$

Then the weight of asset A in (B.4) can be rewritten as:

$$
w_{A,t} = \frac{\mathbb{E}[P_{A/B,T|t}|P_{A/B,T|t}>1]}{\mathbb{E}[P_{A/B,T|t}|P_{A/B,T|t}>1]+\mathbb{P}[P_{A/B,T|t}<1]}\,. \tag{B.9}
$$

The weight of asset B is:

$$
w_{B,t} = \frac{\mathbb{P}(P_{A/B,T|t}<1)}{\mathbb{E}[P_{A/B,T|t}|P_{A/B,T|t}>1]+\mathbb{P}[P_{A/B,T|t}<1]}\,. \tag{B.10}
$$

Appendix C. Sensitivity of $w_{A,t}$ in the special case where $P_{A/B,t|t_0} = 1$

In the special case where $P_{A/B,t|t_0} = 1$, we have $d_1 = -d_2$ and $\Phi(d_1) = 1 - \Phi(d_2)$. From (3), the weight of the high–risk asset is then:

$$
w_{A,t} = \frac{P_{A/B,t|t_0}\Phi(d_1)}{P_{A/B,t|t_0}\Phi(d_1)+(1-\Phi(d_2))} = \frac{\Phi(d_1)}{\Phi(d_1)+\Phi(d_1)} = \frac{1}{2}\,. \tag{C.1}
$$

Appendix D. Sensitivity of $w_{A,t}$ with respect to $P_{A/B,t|t_0}$

The partial derivative of $\Phi(d_i)$ $(i = 1, 2)$ with respect to $P_{A/B,t|t_0}$ is:

$$\frac{\partial \Phi(d_i)}{\partial P_{A/B,t|t_0}} = \phi\left(d_i\right) \frac{1}{P_{A/B,t|t_0} \sigma_{A-B} \sqrt{\tau}} \geq 0\,. \tag{D.1}$$

Then the partial derivative of $w_{A,t}$ with respect to $P_{A/B,t|t_0}$ is:

$$\begin{aligned}
\frac{\partial w_{A,t}}{\partial P_{A/B,t|t_0}} &= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \times \Bigg[\left(P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right)\Bigg(\Phi(d_1) \\
&+ P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial P_{A/B,t|t_0}}\Bigg) - P_{A/B,t|t_0}\Phi(d_1)\left(\Phi(d_1) + P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial P_{A/B,t|t_0}} - \frac{\partial \Phi(d_2)}{\partial P_{A/B,t|t_0}}\right)\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \times \Bigg[P_{A/B,t|t_0}\Phi^2(d_1) + P^2_{A/B,t}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial P_{A/B,t|t_0}} \\
&+ (1-\Phi(d_2))\Phi(d_1) + P_{A/B,t|t_0}(1-\Phi(d_2))\frac{\partial \Phi(d_1)}{\partial P_{A/B,t|t_0}} - P_{A/B,t|t_0}\Phi^2(d_1) \\
&- P^2_{A/B,t}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial P_{A/B,t|t_0}} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial P_{A/B,t|t_0}}\Bigg] \\
&= \frac{(1-\Phi(d_2))\Phi(d_1) + P_{A/B,t|t_0}(1-\Phi(d_2))\frac{\partial \Phi(d_1)}{\partial P_{A/B,t|t_0}} + P_{A/B,t|t_0}\frac{\partial \Phi(d_2)}{\partial P_{A/B,t|t_0}}}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \\
&= \frac{(1-\Phi(d_2))\Phi(d_1) + (1-\Phi(d_2))\phi\left(d_1\right)\frac{1}{\sigma_{A-B}\sqrt{\tau}} + \phi\left(d_2\right)\frac{1}{\sigma_{A-B}\sqrt{\tau}}}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \\
&= \frac{(1-\Phi(d_2))\Phi(d_1) + \frac{1}{\sigma_{A-B}\sqrt{\tau}}\left[(1-\Phi(d_2))\phi\left(d_1\right) + \phi\left(d_2\right)\right]}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\,.
\end{aligned} \tag{D.2}$$

Expression (D.2) shows the non–negative relation between the relative price and the weight of the high–risk asset in the MBo2 replicating portfolio. The higher the relative price, the higher its weight in the MBo2 replicating portfolio.

Appendix E. Sensitivity of $w_{A,t}$ with respect to σ_A

The partial derivative of $\Phi(d_1)$ with respect to σ_A is:

$$\begin{aligned}
\frac{\partial \Phi(d_1)}{\partial \sigma_A} &= \phi\left(d_1\right)\left[-\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\frac{1}{2}\sqrt{\tau}\left(2\sigma_A - 2\sigma_B\rho_{A,B}\right) + \frac{1}{2}\sqrt{\tau}\left(2\sigma_A - 2\sigma_B\rho_{A,B}\right)\right] \\
&= \phi\left(d_1\right)\sqrt{\tau}\left(\sigma_A - \sigma_B\rho_{A,B}\right)\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right)\,.
\end{aligned} \tag{E.1}$$

And the partial derivative of $\Phi(d_2)$ with respect to σ_A is:

$$
\begin{aligned}
\frac{\partial \Phi(d_2)}{\partial \sigma_A} &= \phi\left(d_2\right)\Big[-\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\frac{1}{2}\sqrt{\tau}\left(2\sigma_A - 2\sigma_B\rho_{A,B}\right) - \frac{1}{2}\sqrt{\tau}\left(2\sigma_A - 2\sigma_B\rho_{A,B}\right)\Big] \\
&= -\phi\left(d_2\right)\sqrt{\tau}\left(\sigma_A - \sigma_B\rho_{A,B}\right)\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right). \qquad \text{(E.2)}
\end{aligned}
$$

Then, the partial derivative of $w_{A,t}$ with respect to σ_A is:

$$
\begin{aligned}
\frac{\partial w_{A,t}}{\partial \sigma_A} &= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[\left(P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right)P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \sigma_A} \\
&- P_{A/B,t|t_0}\Phi(d_1)\left(\frac{P_{A/B,t|t_0}\partial\Phi(d_1)}{\partial \sigma_A} - \frac{\partial \Phi(d_2)}{\partial \sigma_A}\right)\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \sigma_A} + (1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \sigma_A} \\
&- P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \sigma_A} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \sigma_A}\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[(1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \sigma_A} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \sigma_A}\Bigg] \\
&= \frac{P_{A/B,t|t_0}\sqrt{\tau}\left(\sigma_A - \sigma_B\rho_{A,B}\right)}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \\
&\times\left[(1-\Phi(d_2))\phi(d_1)\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right) - \Phi(d_1)\phi(d_2)\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right)\right]. \qquad \text{(E.3)}
\end{aligned}
$$

Appendix F. Sensitivity of $w_{A,t}$ with respect to σ_B

The partial derivative of $\Phi(d_1)$ with respect to σ_B is:

$$
\begin{aligned}
\frac{\partial \Phi(d_1)}{\partial \sigma_B} &= \phi\left(d_1\right)\Big[-\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\frac{1}{2}\sqrt{\tau}\left(2\sigma_B - 2\sigma_A\rho_{A,B}\right) + \frac{1}{2}\sqrt{\tau}\left(2\sigma_B - 2\sigma_A\rho_{A,B}\right)\Big] \\
&= \phi\left(d_1\right)\sqrt{\tau}\left(\sigma_B - \sigma_A\rho_{A,B}\right)\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right). \qquad \text{(F.1)}
\end{aligned}
$$

And the partial derivative of $\Phi(d_2)$ with respect to σ_B is:

$$
\begin{aligned}
\frac{\partial \Phi(d_2)}{\partial \sigma_B} &= \phi\left(d_2\right)\Big[-\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\frac{1}{2}\sqrt{\tau}\left(2\sigma_B - 2\sigma_A\rho_{A,B}\right) - \frac{1}{2}\sqrt{\tau}\left(2\sigma_B - 2\sigma_A\rho_{A,B}\right)\Big] \\
&= -\phi\left(d_2\right)\sqrt{\tau}\left(\sigma_B - \sigma_A\rho_{A,B}\right)\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right). \qquad \text{(F.2)}
\end{aligned}
$$

Then, the partial derivative of $w_{A,t}$ with respect to σ_B is:

$$
\begin{aligned}
\frac{\partial w_{A,t}}{\partial \sigma_B} &= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[\left(P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right) P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \sigma_A} \\
&- P_{A/B,t|t_0}\Phi(d_1)\left(\frac{P_{A/B,t|t_0}\partial \Phi(d_1)}{\partial \sigma_A} - \frac{\partial \Phi(d_2)}{\partial \sigma_A}\right)\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \sigma_B} + (1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \sigma_B} \\
&- P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \sigma_B} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \sigma_B}\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\left[(1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \sigma_B} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \sigma_B}\right] \\
&= \frac{P_{A/B,t|t_0}\sqrt{\tau}\left(\sigma_B - \sigma_A\rho_{A,B}\right)}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \\
&\times \left[(1-\Phi(d_2))\phi(d_1)\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right) - \Phi(d_1)\phi(d_2)\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right)\right].
\end{aligned}
\tag{F.3}
$$

Appendix G. Sensitivity of $w_{A,t}$ with respect to $\rho_{A,B}$

The partial derivative of $\Phi(d_1)$ with respect to $\rho_{A,B}$ is:

$$
\begin{aligned}
\frac{\partial \Phi(d_1)}{\partial \rho_{A,B}} &= \phi(d_1)\left[-\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\frac{1}{2}\sqrt{\tau}\left(-2\sigma_A\sigma_B\right) + \frac{1}{2}\sqrt{\tau}\left(-2\sigma_A\sigma_B\right)\right] \\
&= \phi(d_1)\sqrt{\tau}\sigma_A\sigma_B\left(\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau} - 1\right).
\end{aligned}
\tag{G.1}
$$

And the partial derivative of $\Phi(d_2)$ with respect to $\rho_{A,B}$ is:

$$
\begin{aligned}
\frac{\partial \Phi(d_2)}{\partial \rho_{A,B}} &= \phi(d_2)\left[-\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\frac{1}{2}\sqrt{\tau}\left(-2\sigma_A\sigma_B\right) - \frac{1}{2}\sqrt{\tau}\left(-2\sigma_A\sigma_B\right)\right] \\
&= \phi(d_2)\sqrt{\tau}\sigma_A\sigma_B\left(\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau} + 1\right).
\end{aligned}
\tag{G.2}
$$

Then we can rewrite the sensitivity of $w_{A,t}$ with respect to $\rho_{A,B}$ as:

$$
\begin{aligned}
\frac{\partial w_{A,t}}{\partial \rho_{A,B}} &= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[\left(P_{A/B,t|t_0}\Phi(d_1) + 1 - \Phi(d_2)\right) P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \rho_{A,B}} \\
&\quad - P_{A/B,t|t_0}\Phi(d_1)\left(\frac{P_{A/B,t|t_0}\partial\Phi(d_1)}{\partial \rho_{A,B}} - \frac{\partial \Phi(d_2)}{\partial \rho_{A,B}}\right)\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \rho_{A,B}} + (1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \rho_{A,B}} \\
&\quad - P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \rho_{A,B}} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \rho_{A,B}}\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[(1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \rho_{A,B}} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \rho_{A,B}}\Bigg] \\
&= \frac{P_{A/B,t|t_0}\sqrt{\tau}\sigma_A\sigma_B}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \\
&\quad \times \left[(1-\Phi(d_2))\phi\left(d_1\right)\left(\frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau} - 1\right) + \Phi(d_1)\phi\left(d_2\right)\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right)\right].
\end{aligned}
\tag{G.3}
$$

Appendix H. Sensitivity of $w_{A,t}$ with respect to τ

The partial derivative of $\Phi(d_1)$ with respect to τ is:

$$
\begin{aligned}
\frac{\partial \Phi(d_1)}{\partial \tau} &= \phi\left(d_1\right)\left[-\frac{\ln(P_{A/B,t|t_0})}{2\sigma_{A-B}\sqrt{(\tau)^3}} + \frac{1}{2}\sigma_{A-B}\sqrt{\tau}\right] \\
&= \phi\left(d_1\right)\frac{1}{2}\sqrt{\tau}\sigma_{A-B}\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right).
\end{aligned}
\tag{H.1}
$$

And the partial derivative of $\Phi(d_2)$ with respect to τ is:

$$
\begin{aligned}
\frac{\partial \Phi(d_2)}{\partial \tau} &= \phi\left(d_2\right)\left[-\frac{\ln(P_{A/B,t|t_0})}{2\sigma_{A-B}\sqrt{(\tau)^3}} - \frac{1}{2}\sigma_{A-B}\sqrt{\tau}\right] \\
&= -\phi\left(d_2\right)\frac{1}{2}\sqrt{\tau}\sigma_{A-B}\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right).
\end{aligned}
\tag{H.2}
$$

Then, the partial derivative of $w_{A,t}$ with respect to the time to maturity of the Margrabe option is:

$$
\begin{aligned}
\frac{\partial w_{A,t}}{\partial \tau} &= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[\left(P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right) P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \tau} \\
&- P_{A/B,t|t_0}\Phi(d_1)\left(\frac{P_{A/B,t|t_0}\partial \Phi(d_1)}{\partial \sigma_A} - \frac{\partial \Phi(d_2)}{\partial \tau}\right)\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \tau} + (1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \tau} \\
&- P^2_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_1)}{\partial \tau} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \tau}\Bigg] \\
&= \frac{1}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2}\Bigg[(1-\Phi(d_2))P_{A/B,t|t_0}\frac{\partial \Phi(d_1)}{\partial \tau} + P_{A/B,t|t_0}\Phi(d_1)\frac{\partial \Phi(d_2)}{\partial \tau}\Bigg] \\
&= \frac{\frac{1}{2}P_{A/B,t|t_0}\sqrt{\tau}\sigma_{A-B}}{\left[P_{A/B,t|t_0}\Phi(d_1) + (1-\Phi(d_2))\right]^2} \\
&\times \left[(1-\Phi(d_2))\phi(d_1)\left(1 - \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right) - \Phi(d_1)\phi(d_2)\left(1 + \frac{\ln(P_{A/B,t|t_0})}{\sigma^2_{A-B}\tau}\right)\right].
\end{aligned}
\tag{H.3}
$$