FINVEX WHITE PAPER ON DYNAMIC RISK BASED ASSET ALLOCATION

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August 2012

Abstract

In asset allocation, the equal risk contribution (ERC) approach gains rapidly in popularity: portfolio weights are dynamically set such that the asset classes contribute equally to the portfolio risk. As such, the portfolio loads automatically less on the more risky asset and, by diversifying across asset classes, portfolio drawdowns are reduced.

When the equity subportfolio is allocated based on market capitalization weights, the ERC portfolio allocation tends to by dominated by the bond allocation. While the resulting portfolio has a low risk, it is in today's market regime of low interest rates, also characterized by only moderate expected returns. For this reason, American fund managers have advocated to overlay the ERC portfolio with leverage. Such a leveraged risk-diversified portfolio is commonly referred to as a risk-parity strategy.

This paper explores two leverage-free alternatives to increase tactically the equity allocation and preserve a completely risk-based asset allocation approach, which dynamically adjusts to the changing risk regimes. The first one consists in replacing the market capitalization weighted equity index with a low risk equity index. The second suggestion to increase the equity allocation is to deviate from an equal risk contribution objective to a 60/40 risk allocation strategy. We show that these two modifications increase significantly the weight of the equity portion in the portfolio, while still keeping the total risk of the dynamically rebalanced portfolios at a low level.

Introduction

Already in 1980, Merton emphasized the relative difficulty in forecasting properly expected returns, versus forecasting risk. Optimisation techniques are well known to be sensitive to differences in expected returns and as a consequence portfolio weights are dominated by the asset classes for which the estimation error in the expected returns is the largest (e.g. Best and Grauer, 1991). In contrast, risk is highly predictable, explaining the recent shift in the investment paradigm from return-based to risk-based portfolio allocation techniques and optimisation models.

Risk-based portfolio solutions are portfolio allocation techniques that do not require explicit modelling of expected returns. Because of the time-variation in risk and dependence across stock returns, optimal risk-based portfolios are rebalanced frequently, such that the ex post risk profile does not drift away from the investment objectives. Examples are the minimum risk portfolio, the 60/40 portfolio and the equal risk contribution portfolio.

In the context of a bond-equity portfolio, invested in the market capitalization weighted equity index, this leads typically to portfolios that have very high allocations to bonds, and given today's low interest rate regime and high bond prices, relatively low expected returns. For this reason, American fund managers have advocated to overlay the equal risk contribution portfolio with leverage. Such a leverage risk-diversified portfolio is commonly referred to as a risk-parity strategy.

This paper emphasizes that a more risk-controlled way of increasing the allocation to equity is to invest in a low risk equity subset when targeting a portfolio where all asset classes are weighted according to an equal risk contribution objective.

A more direct way of increasing the equity portion in the portfolio is to tactically decide to switch from the equal risk contribution objective to a 60/40 risk allocation objective. In contrast to the 60/40 weight allocation portfolio, the 60/40 risk allocation portfolio loads automatically less on the more risky asset and has therefore still a low overall risk.

In what follows, we first review the main theoretical aspects of equal risk contribution portfolios. Then the empirical properties of low risk equity portfolios are compared with those of market cap weighted indices, both in a stand-alone case as in a context of bond-equity portfolio allocation.

Review on equal risk contribution portfolios

A major advantage of a portfolio invested in multiple asset classes is that the portfolio risk can be diversified across the different asset risk factors. As such, portfolio drawdowns are reduced compared to a portfolio that is invested in the most risky asset, while attractive upward return potential is created, compared to investing in the asset class with the lowest risk. The highest diversification of risk is obtained by the equal risk contribution (ERC) portfolio, in which all assets contribute equally to portfolio risk.

The risk contribution of an asset is formally defined as the simple product of its portfolio weight and its marginal contribution to risk. For simplicity in exposition, we focus here on the portfolio standard deviation as risk measure.1 Qian (2006) illustrates that for many common cases these risk contributions can be directly linked to the asset's loss contribution. Furthermore, the volatility risk contribution has a simple explicit expression. More precisely, let Σ denote the covariance matrix for the return over the reference period. Then the risk contribution of the ith asset in a portfolio with weights w is given by:

$$RC_i = w_i \Sigma w / \sqrt{w' \Sigma w}.$$

Because of the 1-homogeneity of standard deviation as a risk measure, Euler's theorem on homogeneous functions implies that the sum of the component risk contributions is the portfolio volatility:

$$\sum_{i=1}^{N} RC_i = \sqrt{w' \Sigma w},$$

where N denotes the number of portfolio components.

Note that, in the general case, the ERC portfolio allocation depends on the relative differences in volatility and correlation of the portfolio components. In fact, as shown by Lee (2011), the higher the volatility and/or the correlation of an asset with other assets, the lower is its weight in the ERC portfolio.

In a 2-asset framework, such as the bond-equity portfolio, the correlation cancels out in the ERC constraint RCBONDS = RCEQUITY and the allocation to equity is exactly equal to the ratio of bond volatility to the sum of equity and bond volatility:

$$w^*_{EQUITY} = \sigma_{EQUITY} / (\sigma_{EQUITY} + \sigma_{BONDS}).$$

This is a key equation, since it translates directly the volatility of the equity portfolio of the portfolio into the portfolio weight. When stock volatility is twice that of bonds, 1/3 of the portfolio will be allocated to stocks.

The ERC criterion is thus an explicit rule for deciding on how to rebalance the portfolio in response to changes in the covariance matrix. This is a distinctive factor of the allocation strategies with respect to the

[1] See Boudt, Carl and Peterson (2012) for an extension to downside risk measures.

more traditional asset allocation strategies that rebalance the portfolio in response to changing market conditions.2

The ERC bond-equity portfolio is characterized by a 50%-50% risk allocation to equity and bonds. There is a priori no economic reason why the 50%-50% allocation is optimal. We feel that, in today's market of low interest rates and high durations, a tactical shift toward a higher risk allocation to equity is desirable in many regards. We define such risk allocation portfolios using the percentage risk contributions:

$$\%RC_i = RC_i / \sqrt{w'\Sigma w}.$$

In analogy with the 60/40 equity-bond weighted allocation, we relax the 50-50 constraint of the ERC portfolio and study below also the 60-40 risk allocation portfolio, where the weight of equity is determined such that equity contributes to 60% of the portfolio volatility.

[2] Constant-proportion portfolio insurance (CPPI) and option-based portfolio insurance (OBPI) are examples of strategies that sell stocks as the market falls and buy stocks as the market rises. Constant-mix strategies buy stocks as the market falls and sell them as it rises (see e.g. Perold and Sharpe, 1988).

ILLUSTRATION

The performance of the ERC portfolio depends clearly on the choice of the underlying equity subportfolio. The most common way of implementing the ERC portfolio is to invest in market capitalization weighted indices. However, in equities, it has been shown by various authors that over the past decades a higher return and a lower risk was achieved by dynamically selecting stocks based on their low risk characteristic rather than their market capitalization. E.g. Clarke, de Silva and Thorley (2006) find that minimum variance portfolios based on the 1000 largest U.S. stocks over the 1968-2005 period achieve a volatility reduction of about 25% while delivering comparable or even higher average returns than the market portfolio.

For the ERC bond-equity portfolio, a direct consequence of investing in a low risk equity subportfolio rather than a market capitalization weighted portfolio, is that the budget allocation to equity increases. Next we illustrate this for a market capitalization weighted and minimum variance index invested in a broad worldwide universe.3 The cumulative value of 100 EUR invested in each of these indices is shown below.

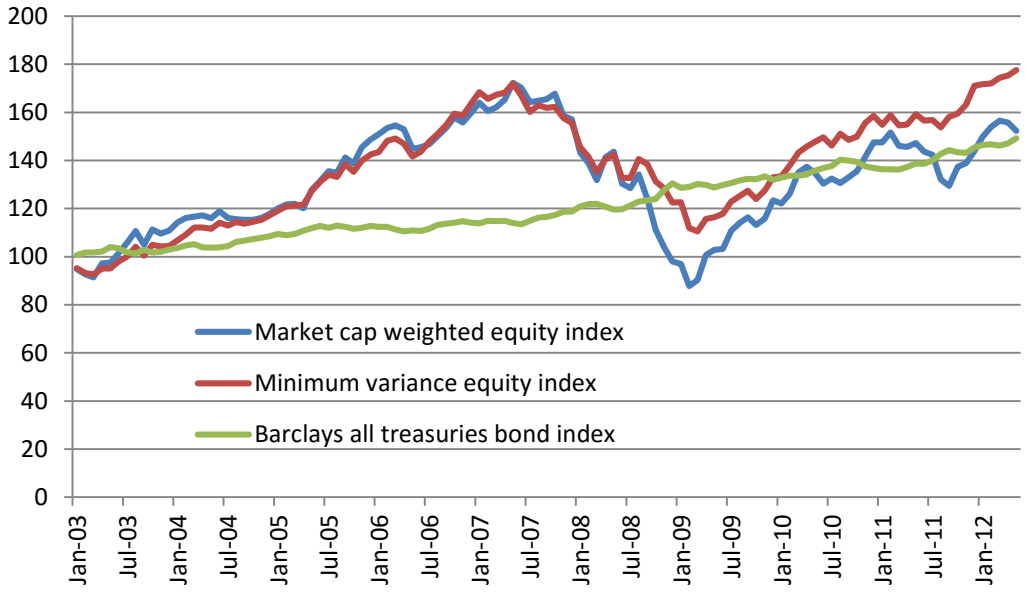

Figure 1 Cumulative value of a buy-and-hold strategy invested in the market cap weighted equity index, the minimum variance equity index and the hedged Barclays global treasuries index.

[3] We choose these indices for illustrative purposes only. The minimum variance index has only a partial approach to risk, as it only minimizes the variance of the equity portfolio. Because of the non-normality of the return distribution, a truely low risk portfolio requires to take into acount the whole return distribution. See Danielsson (2011) for more theoretical details and Boudt and Peeters (2012) for the practical application. Because of the time-variation in risk, the minimum variance portfolio is rebalanced on a regular basis.

Consistent with the literature on the low volatility anomaly (e.g. Baker et al, 2011, Boudt and Peeters, 2012) the minimum variance index offers over the January 2002 – May 2012 a higher return (6% versus 5%) and a lower standard deviation (9% versus 13%). Over the same period, a buy-and-hold investment strategy in the Barclays Global Treasury bond index yielded an annualised compound return of 4%, with an annualised standard deviation of 3%.

A dynamic equal risk contribution portfolio involves estimating on a rolling basis the volatilities of the bond and equity subportfolios, and adjusting the portfolio weights such that they contribute equally to the overall portfolio risk. In our example, we measure volatility using a rolling one year historical volatility estimate computed from daily returns over the period January 2002 – May 2012 and assume a monthly rebalancing frequency.[4] The rolling volatility estimates are plotted in Figure 2.

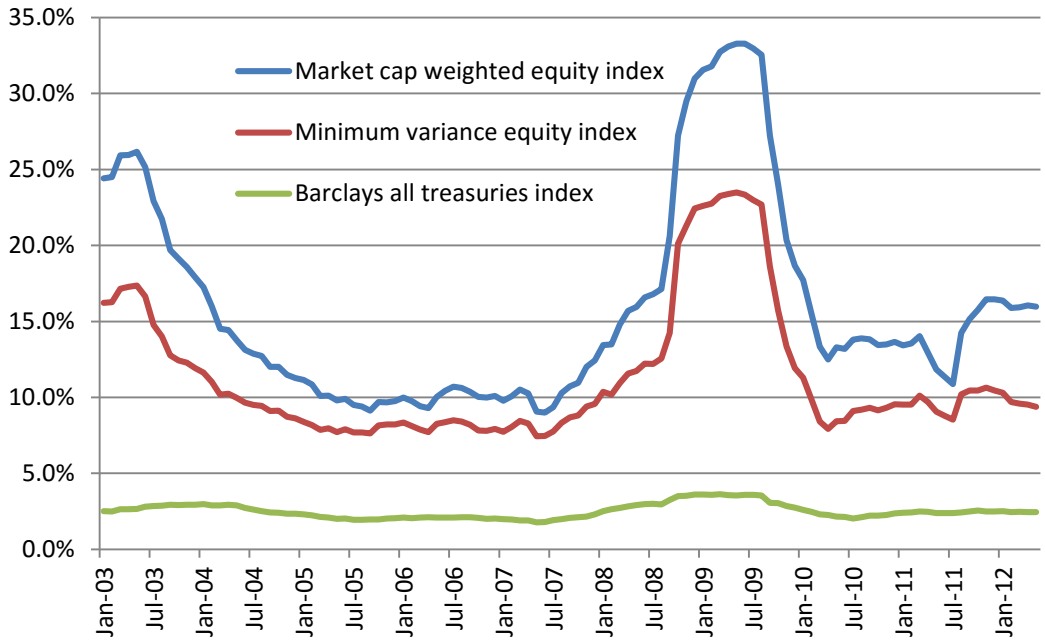

Figure 2 One-year rolling volatility estimates for the market cap weighted and minmum variance equity index, and the hedged Barclays global treasuries index.

Figure 3 shows the sensitivity of the equity weights to the dynamic asset allocation rule. The blue line is the reference line corresponding to the weight of equity in the ERC portfolio, when the equity subportfolio is allocated in function of the market capitalization of stocks. By design, the weight of equity in the

[4] Such a simple risk estimate was chosen for expositional purposes and also because of the short horizon. More accurate estimates of long term bond risk can be constructed using the bond duration and potentially also the volatility of credit default swaps, as in Bruder et al (2011).

portfolio increases, when the relative volatility of equity decreases and vice versa. Over the whole period, the volatility of the market cap weighted index was between 4 and 10 times the volatility of the bond index, leading to equity weights in the ERC portfolio that vary between 9 and 18%, with an average value of 15%.

Over that same period, the minimum variance equity has a substantially lower volatility than the market capitalization weighted index. Consequently, substituting the market cap index with the minimum variance index leads to a significant upward shift of the equity allocation in the ERC portfolio. When invested in the minimum variance equity index, the allocation of equity varies between 13 and 23%, with an average value of 19%.

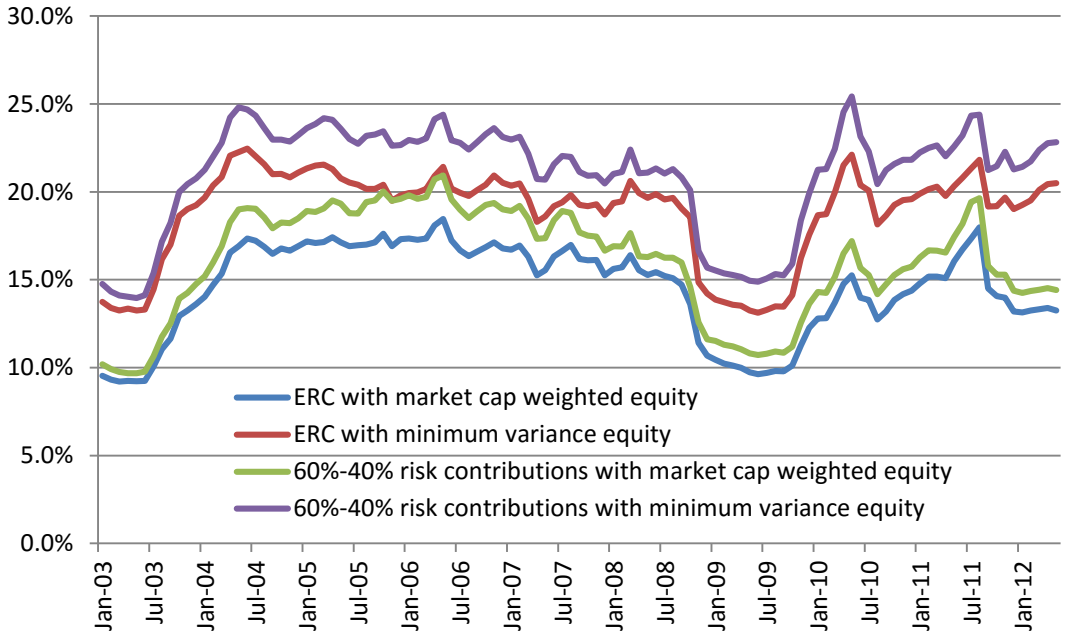

Figure 3 Weight of equity in the ERC and 60-40 risk allocation portfolio. The equity portion is either invested in the world market cap weighted index or the minimum variance index.

Figure 3 shows also the sensitivity of the equity weights to changing the equal risk contribution constraint to a 60%-40% equity-bonds risk allocation constraint. When the market cap weighted index is used, this leads to an increase of the average equity weight from 15% to 16%. For the minimum variance index, the average weight increases from 19% to 21%.

In Table 1 we analyse the impact on the monthly returns of switching from a single asset class strategy to a dynamic risk allocation based strategy.

We observe that, for both the market cap weighted and minimum variance indices, the dynamic ERC or 60/40 risk allocation strategy improves substantially the risk adjusted performance of the portfolio. This is not the case for the constant mix 60% equity, 40% bonds portfolio, for which the Sharpe ratio and max drawdown is significantly worse than investment in the bond index.

Over the 2003-2012 evaluation period, the minimum variance index had the highest return, at the price of a significantly higher risk compared to investing in bonds. An attractive compromise of both is obtained by combining the minimum variance equity and bond index in the ERC or 60/40 risk weighted portfolios. Because of the combination with bonds, the maximum drawdown of the ERC and 60/40 risk weighted portfolios is reduced from 35.7% to less than 3% (compared to 49% for the market capitalization weighted index).

Table 1 Monthly returns analysis of portfolios invested in the world market capitalization weighted index and minimum variance index, as well as the hedged Barclays all treasuries bond index.

	Annualized return	Annualized standard deviation	Sharpe ratio (RF=0)	Max drawdown
Single-asset class strategies				
Hedged Barclays all treasuries bond index	4.3%	2.9%	1.486	-3%
Market cap weighted equity	4.6%	13.2%	0.345	-49.0%
Minimum variance equity	6.3%	9.4%	0.668	-35.7%
Dynamically rebalanced portfolios				
ERC with market cap weighted equity	4.4%	2.5%	1.777	-2.4%
ERC with minimum variance equity	4.9%	2.7%	1.805	-2.6%
60% equity risk contribution, 40% bond risk contribution with market cap weighted equity	4.4%	2.6%	1.712	-2.5%
60% equity risk contribution, 40% bond risk contribution with minimum variance equity	5.0%	2.8%	1.804	-2.9%
Constant mix portfolio with 60% market cap weighted equity, 40% bonds	4.7%	7.6%	0.628	-29.1%
Constant mix portfolio with 60% minimum variance equity, 40% bonds	5.6%	5.6%	1.015	-18.6%

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