Smart beta equity investing through calm and storm

Kris Boudt[a,b] Joakim Darras[c] Giang Ha Nguyen[a] Benedict Peeters[c]

[a]Solvay Business School, Vrije Universiteit Brussel, Pleinlaan 2, 1050 Brussels, Belgium.

[b]VU University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands.

[c]Finvex Group, Arenbergstraat 44, 1000 Brussels, Belgium.

Abstract

Smart beta portfolios typically achieve a superior diversification than the benchmark market capitalization weighted portfolio, but remain vulnerable to broad market downturns. We examine tactical investment decision rules to switch timely between equity and cash investments based on an underlying regime-switching model with macro-economic, macro-financial and smart-beta momentum variables as drivers for the time-varying transition probabilities. A regression-based method is applied to select the relevant state variables. An extensive out-of-sample evaluation for the S&P 500 stocks over the period 1991-2014 shows the gains of smart beta portfolios, the usage of time-varying transition probabilities and the implementation of a dynamic minimum expected return constraint which is more restrictive in bearish markets than in up trending markets. The resulting investment decisions are more reactive to changes in the market conditions and lead to a substantially better risk-adjusted performance and lower drawdowns.

1. Introduction

Tobin's separation theorem recommends investors to combine an investment in the maximum Sharpe ratio risky asset with an appropriate amount of cash. Based on the CAPM model, the investment community has traditionally put forward the market capitalization weighted portfolio as maximum Sharpe ratio portfolio. This traditional choice is increasingly criticized because the underlying CAPM assumptions are often not met on real data. At the same time, a growing literature on smart beta portfolios shows that risk-based portfolios outperform the market capitalization weighted portfolio on the long run in terms of risk adjusted return (Baker et al. (2011)).

Smart beta portfolios typically achieve a superior diversification than the benchmark market capitalization weighted portfolio, but remain vulnerable to broad market downturns. This paper proposes a pure quantitative tactical asset allocation framework that removes emotion and subjective decision-making by delegating the allocation decision to a simple rule: Invest as long as the market is expected to be rising or flat and the estimated downside risk is below its maximum risk level. The predicted return and expected shortfall are computed under a regime switching model with an optimized set of macro-economic, macro-financial and equity momentum variables driving the dynamic transition probabilities. The out-of-sample analysis examines the performance of the investment rules on the universe of S&P 500 constituents over the period 1991-2014 and compares the performance when the underlying equity strategy is the standard market capitalization weighted portfolio with the alternative use of three smart beta equity investments, namely inverse volatility weighted, equally weighted and fundamental value weighted portfolios invested in the S&P 500 constituents.

The proposed tactical allocation framework is closely linked to Faber's (2007) market timing model. It is founded on the time-tested intuition that market timing based on trend-following strategy is a risk-reduction technique that signals when an investor should exit a risky asset class in favor of risk-free investments. Market timing can have a major impact on reducing volatility, avoiding large negative returns and (because of the asymmetric impact of large negative and positive returns on compound performance) it tends to lead to a higher long term investment value. Similar results are obtained by Kritzman et al. (2012) in an asset allocation framework, where they use a regime switching model for state variables and show that a tactical asset allocation strategy based on differentiating investments in high and low regimes of the state variables (e.g. financial stability versus financial turbulence, high and low inflation, economic growth versus recessions) leads out-of-sample to higher investment returns, lower volatility, lower Value at risk (VaR) and a lower maximum drawdown, compared to constant mix strategies.

The proposed market timing model with downside risk control requires a forward looking mean and risk estimate, which we obtain under parametric assumptions on the return generating process. An appropriate model for equity returns needs to accommodate the stylized facts of time-varying volatility, skewness and kurtosis of stock returns. The workhorse method in applied financial time series is to use rolling estimation samples to accommodate the gradual changes in the return distribution. Such a method is however inherently slow in adapting to abrupt changes in the return distribution. These may happen when the economy's endowment switches between high and low economic growth (Cecchetti et al. (1993)), in case of asset pricing bubbles and collapses (Blanchard and Watson (1982)) or in times of transitions between exogenous and endogenous risk regimes (Danielsson (2011)). In order to account for these sudden changes in the return distribution, we will use Hamilton's (1989) regime switching model in which, conditionally on the regime, the stock return is normally distributed. As shown by Guidolin and Timmermann (2008), such a regime switching model is able to accommodate fat tails and skewness in the return distribution, and since it is estimated on rolling estimation samples, it is also robust with respect to the more persistent volatility dynamics observed in return data.

In addition to the across-sample dynamics in the model parameters captured by the rolling sample estimation, we expect that the transition probabilities of the regime switching model change within the sample in function of the changing market conditions. We follow the standard approach to generate dynamics in the transition probabilities by using lagged variables as the source of time variation (Diebold et al. (1994); Schaller and Van Norden (1997)). These authors use one state variable. We will take a composite of state variables that we obtain based on an underlying linear regression model to forecast returns and where variable selection techniques are used to determine the relevant variables. The fitted return (a linear combination of the factors) is used as the state variable to drive the time-variation in the transition probabilities.

Given the predicted return and risk under the proposed regime switching mean-variance model for the smart beta equity returns, we then study the performance of the (risk controlled) market timing investment rules.

Our first finding is that for the S&P 500 stocks over the period 1991-2014, the inverse volatility weighted portfolio offers the best risk-adjusted performance of all smart beta portfolios considered. This ranking is robust to the analysis that controls for the Fama-French-Carhart factor exposures, since we find that the alpha is the highest for the inverse volatility weighted portfolio. Second and third best in terms of alpha is the equally weighted portfolio and fundamental value weighted portfolio, while the market capitalization weighted portfolio thus has the worst alpha of all portfolio weightings considered.

However, all pure equity portfolios have large drawdowns that range between 36% for the inverse risk weighted portfolio and 58% for the fundamental value weighted portfolio. We find that the active strategy of switching between the equity portfolio and the cash position improves substantially the largest drawdown, which now ranges between 15% and 20%. It further improves also the other performance measures: the switching portfolios have higher return, significantly lower volatility and lower downside risk than their equity-only counterparts.

The remainder of chapter is organized as follows. Section 2 presents the regime switching model based approach to market timing the smart beta equity investment. Section 3 describes the sample and the variables used in the analysis. Our findings are presented in Section 4. Finally, Section 5 concludes and sketches directions for further research. The appendix contains technical details on the implementation of the regime switching investment model.

2. A regime switching approach to market timing

Faber (2007) shows the good performance of a simple, but effective investment rule based on a trend signal extracted from rolling prices. We first review this approach and then present an alternative investment decision framework based on a regime switching model for the risky asset returns. We limit ourselves to the equity timing decision and do not consider approaches (such as CPPI investments) that aim to construct an optimally weighted portfolio of cash and equity investment.

2.1. Faber's timing model based on rolling price averages

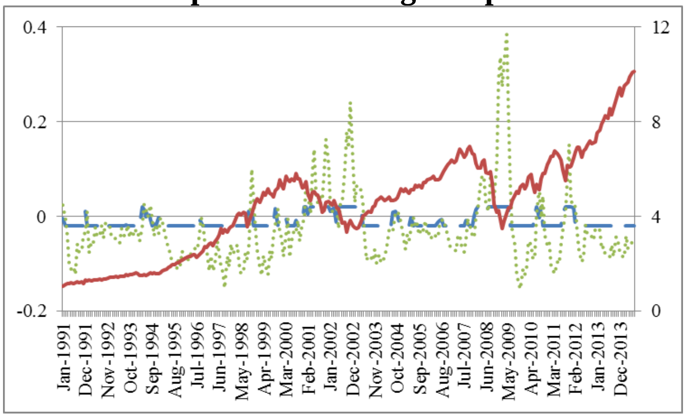
Faber (2007) evaluates in detail the performance of a trend-following market timing model. He emphasizes that, in order to avoid behavioral bias in investment decisions, a quantitative investment model is needed. He recommends a trend-following model which invests in a risky asset over the period [t-1, t] when the price of the risky asset at time t-1 is greater than the 10-month simple moving average. The condition that P_{t-1} exceeds the simple average of P_{t-1}

P_{t-2},…,P_{t-10} can be reverse engineered to find the condition on the asset return in month t-1 needed for this condition to be satisfied. Denote this required return as κ_{t-1}. It can be shown that the condition that P_{t-1} exceeds the simple average of P_{t-1} P_{t-2},…,P_{t-10} is equivalent to requiring that the asset return in month *t-1* exceeds the following dynamic threshold:

$$\kappa_{t-1} = \frac{\frac{1}{9}\sum_{i=2}^{10} P_{t-i}}{P_{t-2}} - 1. \qquad \textbf{(1. 1)}$$

Otherwise, the investment is in cash. This dynamic threshold is shown in the short-dashed line in Figure 1. We see that in bullish markets, the threshold becomes negative, while in bearish markets the threshold becomes relatively high and the condition that r_{t-1} exceeds κ_{t-1} becomes thus very restrictive. By lowering the threshold in bullish equity markets, the probability of investing in equities increases, while vice versa in bearish markets. This explains why the Faber strategy is qualified as a trend following investment strategy. Note in Figure 1 that the dynamic threshold κ_{t-1} of Faber (2007) can become relatively extreme.

Figure 1: The trend following minimum return targets $\boldsymbol{\kappa}$ and $\tilde{\boldsymbol{\kappa}}$ when the corresponding equity investment is the market capitalization weighted portfolio



Note: *On the right-hand side, the full line is the cumulated return of the market capitalization weighted portfolio; on the left-hand side, the short-dashed line presents the required return rate as Faber's strategy and the long-dashed line presents the trend following return target.*

The advantage of Faber's momentum strategy is that it is also a risk management model. By avoiding the market downtrends, a significant reduction in volatility is achieved. Faber (2007) shows that the performance gains in terms of lower volatility and higher return are not a result of data mining, since they are found for different markets including stocks, bonds, commodities and real estate market, and over many time periods.

The disadvantage of Faber's momentum strategy is that it is too mechanical and does not exploit the information in state variables predicting the future return distribution. A solution for this is our timing model based on the predicted return distribution from a mean-variance regime switching model with state variables driving transition probabilities presented in the next section.

A further drawback is that the investment decision is backward-looking (invests in equity over the horizon [t-1, t] if the past return r_{t-1} exceeds the market trend following threshold κ_{t-1} defined in (1.1)). We will consider a forward-looking alternative, which invests in equity when the predicted return for the period [t-1, t] ($\mu_{t|t-1}$) is such that the predicted stock price exceeds the (predicted) 10-month simple moving average, i.e. that $P_{t-1}(1+\mu_{t|t-1})$ exceeds the simple average of $P_{t-1}(1+\mu_{t|t-1})$, P_{t-1},…, P_{t-9}. In order to avoid extreme conditions, the threshold is truncated at -2% and +2%. As such risky equity investments are avoided when the predicted return is less than -2%, while the equity investment is enforced when the predicted return is above 2%. More precisely, we invest in equities when the predicted return $\mu_{t|t-1}$exceeds the trend following return target threshold $\tilde{\kappa}_{t-1}$ given by:

$$\tilde{\kappa}_{t-1} = \min\{\max\{\frac{\frac{1}{9}\sum_{i=1}^{9} P_{t-i}}{P_{t-1}} - 1, \text{-2\%}\}, \ 2\%\}. \qquad \textbf{(1. 2)}$$

The resulting time series of trend following return target is plotted in the long-dashed line in Figure 1. We see that during the bullish market, it usually stays at -2% and it is close to 2% in bearish markets.

2.2. Timing model based on the predicted return distribution from a mean-variance regime switching model with state variables driving the transition probabilities

We first introduce the model in Subsections 2.2.1-2.2.2 and then discuss the investment decision process in Subsection 2.2.3.

2.2.1. Regime switching model for equity returns

Let y_t be the monthly stock return (t = 1, 2, ..., T) whose distribution depends on a state variable S_t. Assume further that there are two states (S_t=1 for the first regime, S_t=2 for the second regime), and that, conditionally on the state S_t, the return distribution is normal with mean μ_{S_t} and variance $\sigma^2_{S_t}$:

$$y_t = \mu_{S_t} + \varepsilon_t; \ \varepsilon_t \sim iid \ N(0, \sigma^2_{S_t}). \qquad \textbf{(1. 3)}$$

The regimes can be interpreted as good and bad regimes. Note that, even though, conditionally on the regime, returns are normal, the unconditional distribution is often non-normal (Ang and Timmermann (2012), Marron and Wand (1992)).

We follow Hamilton (2008) and assume the states to be unobserved. Likelihood-based filters are used to infer the state S_t from the observed y_t's under the additional assumption that the latent state variable S_t is a realization of a Markov chain with time-varying transition probabilities:

$$\Pr(S_t = \text{j}|S_{t-1} = \text{i}, S_{t-2} = \text{k},\ldots) = \Pr(S_t = \text{j}|S_{t-1} = \text{i}) = p_{ij,t}. \qquad \textbf{(1. 4)}$$

The specification in (1.4) assumes that the probability of a change in regime depends on the past only through the most recent regime. The properties of $p_{ij,t}$ are: $\sum_{j=1}^{2} p_{ij,t} = 1$; and $p_{ij,t} \geq 0 \ \forall i, j \ \in \{1,2\}$.

The diagonal elements of this matrix are parameterized using the logit transformation of an underlying time-varying process $\pi_{1,t}, \pi_{2,t}$ (Diebold et al. (1994)):

$$p_{11,t} = \exp(\pi_{1,t}) / [1 + \exp(\pi_{1,t})] \qquad \textbf{(1. 5)}$$

$$p_{22,t} = \exp(\pi_{2,t}) / [1 + \exp(\pi_{2,t})], \qquad \textbf{(1. 6)}$$

where the time-variation in the transition probabilities is driven by the state of the financial and economic system, as described by a state variable *x*:

$$\pi_{1,t} = c_1 + d_1 x_{t-1} \qquad \textbf{(1. 7)}$$

$$\pi_{2,t} = c_2 + d_2 x_{t-1}. \qquad \textbf{(1. 8)}$$

The parameters c_j and d_j are the intercept and coefficient modeling the impact of the lagged value of the state variable on the time-varying transition probability for state j (j= 1, 2). Of course, from them, we have $p_{12,t} = 1 - p_{11,t}$, and $p_{21,t} = 1 - p_{22,t}$.

Table 1 shows the state variables and the related literature that we use to construct a composite variable, defined as the predicted return of the following linear prediction model for stock returns:

$$r_t = \beta_0 + \sum_{i=1}^{N} \beta_i V_{i,t-1} + \epsilon_t, \qquad \textbf{(1. 9)}$$

where the variables are a subset of those in Table 1, selected as the subset having the lowest BIC. The predicted return under this selected model is used as the driver for the transition probabilities, both in-sample and out-of-sample to forecast return and transition probabilities for the next period.

2.2.2. Estimation

The model parameters are estimated by maximum likelihood on rolling estimation samples of six years of monthly observations. For each estimation window, 8 parameters are estimated by maximum likelihood techniques (the parameter vector $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, c_1, c_2, d_1, d_2)$ where μ_j and σ_j are the mean and volatility of the risky asset return in each regime, and c_j and d_j are the intercept and slope coefficient in time-varying transition probability matrix, respectively; *1, 2* stands for the good and bad regime).[1] The likelihood is calculated under the weighted likelihood approach in which the more recent observations receive a higher weight. The motivation is that relatively long estimation samples are needed to calibrate the regime switching model (6 years) and by using the exponentially decaying weights, the obtained predictions are more robust to the in-sample changes in the parameters. Following Meucci (2013), the weights are defined as follows:

$$w_t = \frac{h_t}{\sum_{t=1}^{\bar{t}} h_t}, \qquad \textbf{(1. 10)}$$

$$h_t = e^{-\frac{\ln(2)}{\tau}|t-\bar{t}|}, \qquad \textbf{(1. 11)}$$

where $\bar{t}$ denotes the most recent observation (*72* in our research) and τ >0 is the half-life of the exponential decay, which we set to 36.

Based on the estimated parameters, the predicted probability that the smart beta return is in each regime in the next period can then readily be computed. Assume $\xi_{i,t-1|t-1}$ to be an inference of probability in regime *i* on date *t-1* based on information up to date *t-1* (obtained using the Hamilton filter, as described in the Appendix), the predicted probability of regime *j* on date *t* is given by:

$$\xi_{j,t|t-1} = \sum_{i=1}^{2} p_{ij,t}\, \xi_{i,t-1|t-1}, \qquad \textbf{(1. 12)}$$

where $p_{ij,t}$ denotes the transition probability from regime *i* to regime *j* on date *t*.

---

[1] The log-likelihood function of the RS model can have local optima. In order to avoid the gradient-based optimization method to be stuck in a local optimum, we first use the stochastic global optimizer Differential Evolution (Price et al. (2006)) to find the best starting value. Based on these starting values, the log-likelihood function is then optimized using the BFGS routine, using transformations to ensure the parameters are well defined.

2.2.3. Investment decision

Our investment decision is based on two factors: predicted return under regime switching model and expected shortfall level. Under the two- regime specification, the predicted return is the weighted expected return of each regime, with weights equal to the predicted probability of each regime:

$$\mu_{t|t-1} = \sum_{j=1}^{2} \xi_{j,t|t-1}\mu_j, \quad \textbf{(1. 13)}$$

where $\xi_{j,t|t-1}$ is the predicted probability of regime j on date t based on the return information available at time *t-1*, and μ_j: mean of return under regime *j*.

The conditional density of the one-step ahead return is a mixture of two Gaussian densities with weights $\xi_{1,t|t-1}$and $\xi_{2,t|t-1}$. We compute the expected shortfall under this mixture distribution, as detailed in the Appendix and Broda and Paolella (2011).

Decision process

In our report, two types of market timing strategies are tested: Faber's market timing rule and the regime switching approach. About Faber's market timing rule, the decision is simply to invest in the equity as long as its price level is greater than the 10 month simple moving average price. Otherwise, a bear market is detected and the portfolio is fully invested in LIBOR.

Under the regime switching approach, the objective is to invest in the risky equity asset as long as the market is expected to be rising or flat, i.e. when $\mu_{t|t-1}$ is above the minimum expected return target. Two types of return target will be tested: i) the fixed return target at -2% and ii) the trend following return target $\tilde{\kappa}_{t-1}$ in (1.2), which is more restrictive in bearish markets ($\tilde{\kappa}_{t-1}$ is around 2%) than in bullish markets ($\tilde{\kappa}_{t-1}$ is around -2%). Otherwise the portfolio is invested in the 1 month London Interbank Offered Rate (LIBOR, in USD).

In addition, we consider the same market timing strategy, but with a downside risk management overlay. As in Yamai and Yoshiba (2005), we restrict ourselves to the equity market as long as the expected market return is above the minimum expected return and the expected shortfall at 95% is less than 10%. Otherwise, the portfolio is invested in the LIBOR. To summarize, we will consider three investment decisions based on the regime switching model (all of them have parameters estimated with the predictive log likelihood in which more recent observations have higher weights):

(i) RS_static: Market timing based on the regime switching model with static transition probabilities;
(ii) RS_dynamic: Market timing based on the regime switching model with time-varying transition probabilities;
(iii) RS_EScontrol: Market timing based on the regime switching model with time-varying transition probabilities and with a control of downside risk (expected shortfall calculated at 95%).

2.3. Performance evaluation

For each time series of out-of-sample monthly return, five performance measures are computed: i) annualized average return; ii) annualized volatility; iii) annualized Sharpe ratio; iv) maximum drawdown; and v) historical expected shortfall at 95%. For the first and second measure, we will test whether the observed differences are statistically significant. For this, we follow Engle and Colacito (2005) by testing significant differences between the monthly portfolio returns and squared returns using a Diebold and Mariano (1995) type test. It regresses the monthly differences between the performance measures of two portfolio methods on a constant, and tests whether the estimated constant is significantly different from zero using a Newey-West standard error. The significance of the difference in Sharpe ratios is evaluated using the test of Jobson and Korkie (1981), Memmel (2003) and Ledoit and Wolf (2008) in which Newey-West standard errors are used to account for the serial correlation and heteroskedasticity in the return series.

For the evaluation of the smart beta portfolios, we further control for style risk by computing the alpha of the portfolios using the Fama-French-Carhart four-factor model to decompose excess returns of the smart beta portfolios into its abnormal return component and the return explained by the exposure to the market, size, value and momentum factors. The estimated abnormal return α (alpha) is the least squares estimation of the intercept in the regression of the excess portfolio return (ER_t) on the four factors in Fama and French (1992) and Carhart (1997):

$$ER_t = \alpha + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \varepsilon_t, \qquad \textbf{(1. 14)}$$

where MKT_t: market excess return on date *t*; SMB_t: size factor on date *t* (i.e. is the average return on the three small portfolios minus the average return on the three big portfolios); HML_t: book-to-market factor on date t (is the average return on the two value portfolios minus the average return on the two growth portfolios) and MOM_t: momentum factor on date t (is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios) (see e.g. Bauer, Koedijk and Otten (2005) and Barber and Lyon (1997)). Data of the four factors are for the U.S. stock market and retrieved from the K. French data library. Note in (1.14) that alpha can typically be interpreted as a measure of out or under-performance relative to market proxy and the size, value and momentum risk factors. Of course, the higher alpha is, the better the performance is. Its statistical significance will be tested using the t-test with HAC standard errors.

3. Sample and variable description

3.1. Choice of risky asset

The investment universe consists of the S&P 500 stocks over the period January-1985 to July-2014. We will consider four types of equity weighting: the traditional market capitalization weighted portfolio, the inverse volatility weighted portfolio (Leote De Carvalho et al. (2012)), the equally–weighted portfolio (DeMiguel et al. (2009)) and the fundamental weighted portfolio (Arnott et al. (2005)). Below we discuss each of them in detail.

Market capitalization weighted portfolio. The market capitalization represents a broadly invested portfolio, which has the advantage of a low turnover and can be interpreted as an equilibrium portfolio (Perold (2007)). The popularity of the market capitalization weighted portfolio originates from the Capital Asset Pricing model (CAPM) which states that the market capitalization weighted portfolio is the maximum Sharpe ratio portfolio under very strict assumptions. In practice, the assumptions are often violated, leading to the observed mean-variance domination of the market capitalization weighted portfolio over long evaluation windows (Baker et al. (2011)).

Inverse volatility weighted portfolio. Among others, Baker and Haugen (2012) and Dutt and Humphery-Jenner (2013) show that low volatility stocks earn higher returns compared to high volatility stocks and they find that this phenomenon is consistent both over time and over different markets. In our application, we follow Leote de Carvalho et al. (2012) and compute the low risk portfolio as the inversely weighted volatility portfolio. The volatilities are estimated over a 252–day moving window. The 100 least volatile stocks are set inversely proportional to the stock's volatility. The same method is applied to construct the S&P 500 Low volatility index (S&P Dow Jones index (2014)).

Equally weighted portfolio. The equally-weighted portfolio represents a naively diversified portfolio, in which all assets (whatever their size, value or risk characteristics) receive the same weight. While the equally-weighted portfolio is highly diversified in terms of weights, the diversification in terms of risk is often limited. Return data series of these portfolios are computed using adjusted price data from COMPUSTAT.

Fundamental value weighted portfolio. Arnott et al. (2005) popularized the approach of fundamental value weighting (also called Fundamental Indexation (FI)). Under this approach, assets are selected and weighted based on fundamental metrics of company size such as book value, number of employees, operating cash flow, dividends, earnings and revenues. Several studies find a long-run outperformance of the FI compared to the market capitalization weighted portfolio (see Walkshausl and Lobe (2010)). In our application, we consider similar fundamental characteristics as Arnott et al. (2005). The fundamental value characteristics are: book value of common equity, dividends, net operating cash flow and sales. We examine a composite fundamental value portfolio in which the weights of the single–metrics are aggregated in equal proportions. Net operating cash flow is taken as the difference between the operating income before depreciation and total accruals (Kothari et al. (2005)). The accrued liabilities at time t is the change in current assets minus the change in cash and short term investments, minus the change in current liabilities excluding long-term debt minus the amount of depreciation and amortization scaled at the lagged value of total assets. The dividends, net operating cash flow and sales measures are five–year rolling averages. The fundamental data are retrieved from the COMPUSTAT database on an annual basis from 1984 to 2014. The metrics are lagged by one quarter to ensure data availability.

3.2. Variables in the multivariate regression model

Table 1 shows the different state variables that we consider as possible drivers for the time-varying transition probabilities. Consistent with previous research of Giot and Petitjean (2011); De Boer and Norman (2014) and many other researchers, we classify them in three groups: macro-economic variables, macro-financial variables and smart-beta momentum variables. The first two groups are exactly the same among smart-beta portfolios, while the last group is the momentum specific to each of them. All variables considered are required to be prespecified and do not depend on the estimated regime switching model. This excludes the use of duration as a driver for time varying transition probabilities (Maheu and McCurdy (2000)). All variables are considered at their end-of-month value. To avoid look-ahead bias, all variables series used in multivariate regression model are always lagged by a month compared to the time series of monthly returns.[2]

Table 1: State variables used in the predictive return model to construct the composite state variable driving the time-variation in the transition probabilities

Variable description	References
Macro-economic variables	
Real GDP growth: y-o-y change of real GDP;	De Boer and Norman (2014)
Inflation: y-o-y change of Consumer Price Index (CPI)	De Boer and Norman (2014); Estrella and Mishkin (1998); Fama and Schwert (1977); Fama (1990);
PMI: Purchasing Manager Index at the end of each month	Estrella and Mishkin (1998); Wolfers and Zitzewitz (2004)

[2] The macro-economic and macro-financial variables are retrieved from the website of the Federal Reserve Bank of St. Louis. The CAY series is downloaded from http://faculty.haas.berkeley.edu/lettau/data_cay.html and the CAPE series from Shiller's website at http://www.econ.yale.edu/~shiller/data.htm

Macro-financial variables	
TED spread: difference between 3-month USD Libor rate and 3-month T-Bill yield	Boudt et al. (2012); De Boer and Norman (2014)
Short term yield: the difference between short term interest rate (3-month T-bill yield) and a 12-month backward-looking moving average	Estrella and Mishkin (1998); Fama (1990); Giot and Petitjean (2011);
Long term yield: the difference between long-term government bond yield (10 year bond) and a 12-month backward-looking moving average;	Estrella and Mishkin (1998); Fama (1990); Giot and Petitjean (2011);
Term spread: Logarithm of difference between long term government bond yield and short term interest rate;	Estrella and Mishkin (1998); Giot and Petitjean (2011); Fama (1990)
Credit spread: difference between Moody's seasoned Baa corporate bond yield and Fred fund rate;	Gilchrist, Yankov and Zakrajšek (2009); Norden and Weber (2009)
VIX: CBOE's implied volatility index of S&P 500 index options;	Boudt et al. (2012); Guo and Whitelaw (2006);
Monthly change of VIX index;	Boudt et al. (2012);
Variance risk premium: The difference between implied variance and realized variance, where implied variance is the squared return of VIX and realized variance is the squared of last three month return of SP500;	Bollerslev et al. (2009)
CAY: Consumption wealth ratio is defined as the log of consumption in the US divided by aggregate wealth;	Lettau and Ludvigson (2001); Bollerslev et al. (2009)
CAPE: Cyclically adjusted price-earnings: price level of the S&P 500 divided by the average of earnings adjusted for inflation;	Taboga (2011)
Fear index: the difference between VIX data and annualized standard deviation of three month daily returns;	Shaikh and Padhi (2014)
Smart-beta momentum variables	
One month return;	Van Dijk and Franses (1999);
Rolling average of three month returns;	Van Dijk and Franses (1999);
Price spread: Difference between the index price and its simple moving average over ten months;	Faber (2007);

The estimation window used for the multivariate regression is also six years, and matches thus with in-sample window of regime switching model. Based on macro-economic data availability of each market, some data series miss data before 1990. Any variable that has a missing observation in the 72-month rolling sample will be omitted. To reduce the impact of outliers, the state variables are winsorized at a lower and upper bound corresponding to their in-sample median +/- two times the median absolute deviation. In each estimation window, the most predictive variables are selected according to the Bayesian Information Criterion (BIC) (Konishi and Kitagawa (2008)). Then the predicted return from the selected regression model is used as state variable for the regime switching model.

4. Results

In this section, firstly, we present the results of the impact of the choice of portfolio weighting method (market capitalization, inverse volatility, equal and fundamental value weighting schemes) on the out-of-sample performance on the universe of S&P 500 stocks. We will show that these portfolios still suffer from extreme downside risks. Then, we will analyze market timing strategies using LIBOR as a safe haven investment in case the trend and/or risk signals indicate the equity investment is estimated to be decreasing in value and/or is too risky.

Table 2: Out of sample performances of different smart beta portfolios and portfolios applying market timing strategies:

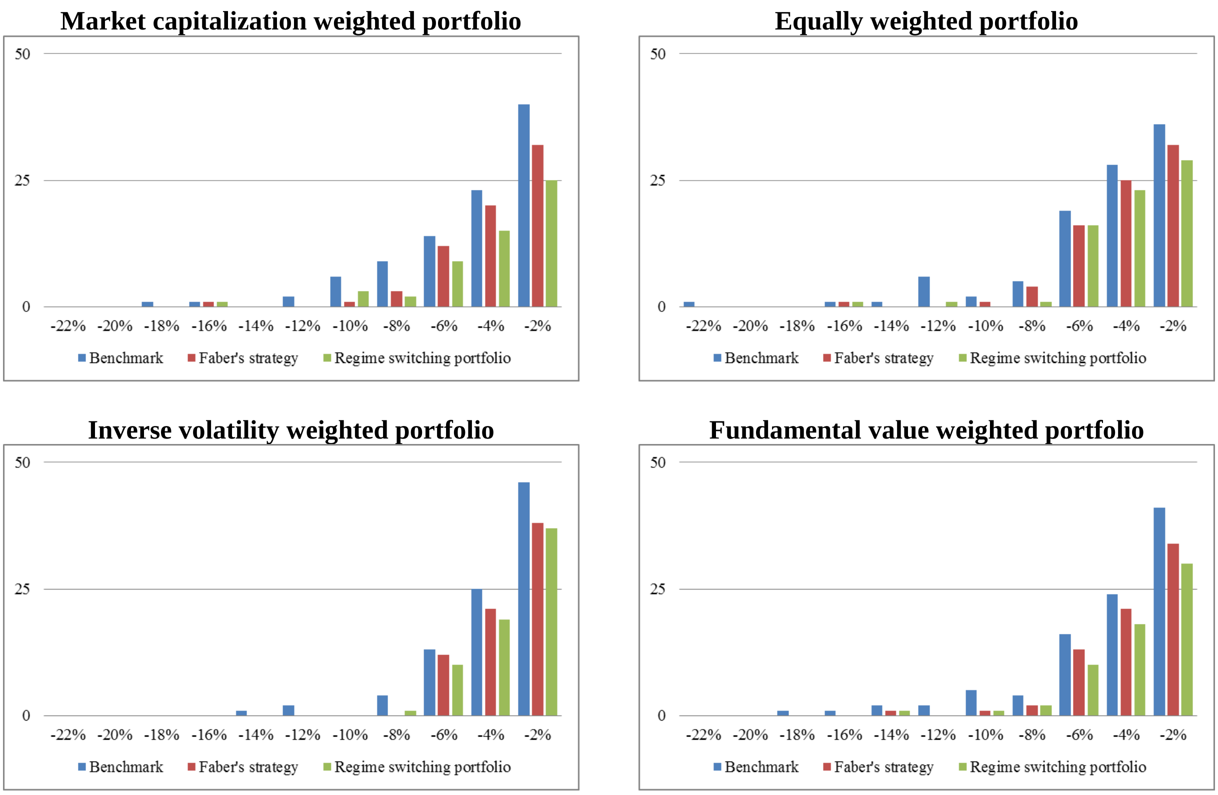
Invest 100% in equity if	Benchmark (buy-and-hold)	Faber's strategy ($r_{t-1} > \kappa_{t-1}$)	RS_static		RS_dynamic		RS_EScontrol	
			$\mu_{t\|t-1} > -2\%$	$\mu_{t\|t-1} > \tilde{\kappa}_{t-1}$	$\mu_{t\|t-1} > -2\%$	$\mu_{t\|t-1} > \tilde{\kappa}_{t-1}$	$\mu_{t\|t-1} > -2\%$	$\mu_{t\|t-1} > \tilde{\kappa}_{t-1}$
(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**
Market capitalization weighted portfolio								
Ann.ret	10.32%	11.13%	9.77%	11.40%	11.45%	11.67%	10.45%	10.71%
Ann.sd	14.53%	***10.41%	13.58%***	***10.65%	***12.11%**	***10.45%	***11.27%	***10.26%
Sharpe	0.71	*1.07	0.72*	*1.07	0.95	**1.12	0.93	1.04
Max.DD	50.05%	17.03%	42.68%	23.72%	32.21%	16.40%	32.21%	16.40%
His.ES	9.30%	5.95%	8.83%	6.16%	7.90%	6.37%	7.38%	6.34%
Num_switch	NA	24	8	28	50	50	42	48
Percentage_in_cash	NA	22.26%	3.89%	20.85%	22.26%	30.39%	31.8%	37.1%
Inverse volatility weighted portfolio								
Ann.ret	11.45%	11.39%	11.45%	10.81%	11.87%	11.72%	12.24%	12.08%
Ann.sd	10.92%	***8.53%	10.92%***	***8.86%*	**10.28%***	***8.42%	**10.12%**	***8.24%
Sharpe	1.05	1.34	1.05	1.22	1.15	*1.39	1.21	**1.47
Max.DD	36.26%	16.22%	36.26%	23.24%	36.26%	15.00%	36.26%	15.00%
His.ES	6.64%	4.46%	6.64%	4.88%	6.18%	4.39%	6.18%	4.38%
Num_switch	NA	26	-	28	22	42	30	50
Percentage_in_cash	NA	17.67%	0%	16.25%	6.01%	20.85%	9.19%	24.03%
Equally weighted portfolio								
Ann.ret	12.94%	11.19%	11.77%	10.71%	14.73%*	13.42%	11.09%	11.74%
Ann.sd	16.51%	***11.47%	15.18%***	***11.66%	**14.12%**	***12.08%	***12.8%	***11.67%
Sharpe	0.78	0.98	0.78	0.92	1.04	1.11	0.87	1.01
Max.DD	55.60%	19.88%	41.47%	19.88%	38.35%	19.88%	29.79%	19.88%
His.ES	10.65%	6.36%	9.79%	6.74%	7.75%	6.10%	7.22%	6.05%
Num_switch	NA	38	12	41	44	61	53	65
Percentage_in_cash	NA	19.43%	3.89%	18.73%	16.25%	25.8%	29.68%	34.98%
Fundamental value weighted portfolio								
Ann.ret	11.73%	11.72%	10.82%	12.02%	12.30%	13.23%	11.29%	12.77%
Ann.sd	15.42%	***10.43%	13.83%***	***10.4%	**12.09%*	***10.54%	***11.42%	***10.28%
Sharpe	0.76	*1.12	0.78	*1.16	1.02	**1.26	0.99	**1.24
Max.DD	58.72%	21.18%	40.86%	15.13%	29.04%	15.13%	32.39%	15.13%
His.ES	10.30%	5.82%	9.07%	5.73%	7.03%	5.64%	6.87%	5.51%
Num_switch	NA	30	8	29	32	36	40	40
Percentage_in_cash	NA	19.43%	3.89%	19.08%	18.37%	25.09%	28.27%	32.51%

*Note: Out of sample period from Jan-1991 to July-2014. ***, **, * on the left hand-side: comparison between market timing portfolios and the benchmark (column 2) at 1%, 5% and 10%, significance level, respectively; ***, **, * on the right hand-side: comparison between the market timing portfolios using regime switching model versus the market timing portfolios using Faber's strategy (column 3) at 1%, 5% and 10% significance level respectively; Expected shortfall (ES) is the historical ES estimate calculated at 95%; Number of switches from risky asset to cash and the reverse (Num_switch) in the out of sample period (283 months); Percentage_in_cash: number of months in cash in the out of sample period; NA: Not applicable.*

4.1. Impact of choice of smart beta portfolio on performance

How does the equity weighting affect the portfolio performance? This is the question we investigate in the second column of Table 2. We find a clear confirmation of the low risk anomaly: the inverse volatility weighted portfolio has a higher return and lower volatility than the market capitalization weighted portfolio (annualized return: 11.45% vs 10.32% and annualized volatility: 10.92% vs 14.53%). Its downside risk (maximum drawdown and historical expected shortfall) is also lower. This can also be seen in Figure 2 where we report the histogram of the portfolio negative returns. We see that the inverse volatility weighted portfolio has more small monthly losses (e.g. [-2%, 0%), [-4%, -2%)) and less extreme monthly losses (e.g. a monthly return below -10%) than the market capitalization weighted portfolio. These results are similar to those reported by Baker et al. (2011), Baker and Haugen (2012) and Leote de Carvalho et al. (2012).

Figure 2: Histogram of monthly negative returns for the equity-only benchmark portfolios, as well as the switching portfolios using rolling price averages (Faber's strategy) and the market timing portfolio using the regime switching model with dynamic transition probabilities and the trend following return target



Note: In the charts, the frequency of the left tail of the return distribution is shown, on the grid [-22%, -20%), [-20%, -18%),… and [-2%,0) respectively. The regime switching portfolios shown in the four charts are those corresponding to market timing based on the RS model with dynamic transition probabilities and the trend following return target (RS_dynamic).

The equally weighted portfolio has the highest annualized return of all four weighting strategies (12.94% p.a). It turns the initial investment of $1 in January 1991 into $17.65 at the end of July 2014, compared to $10.14, $12.88 and $13.68 for the market capitalization weighted, inverse volatility weighted and fundamental value weighted portfolio. In terms of risk, the equally weighted and fundamental value weighted portfolios have a higher volatility than the market capitalization weighted portfolio (16.51% and 15.42% vs 14.53%). Also, their maximum drawdown levels are higher than the one of the market capitalization weighted portfolio (55%-

58% vs 50%). The higher risks of extreme negative returns for these portfolios are illustrated in Figure 2 where we show the histogram of negative returns for the equity-only investment in the first column of each block. We see that extreme losses of equal weight and fundamental value weighted portfolio are more frequent than that of the market capitalization and inverse volatility weighted portfolio.

Table 3: Alpha of smart-beta portfolios calculated from Fama-French-Carhart four factor model

Portfolio	Market cap. weighted	Inverse volatility weighted	Equally weighted	Fundamental value weighted
(1)	**(2)**	**(3)**	**(4)**	**(5)**
Alpha	0.0432	0.2226**	0.1715**	0.1029**
(Se)	(0.0323)	(0.1129)	(0.0682)	(0.0454)

*Note: Alpha and standard error (denoted as Se and put in parentheses) are multiplied by 100. ***, **, * on the right hand-side: significance test with HAC standard error at 1%, 5% and 10%, significance level respectively.*

The return outperformances of smart-beta portfolios are confirmed by the alpha-performance (which corrects for the exposure of the investments with respect to the Fama-French-Carhart four factor model) presented in Table 3. They show that both three smart beta portfolios have significantly positive alpha after adjusting for both four risk factors. Their alphas range from 0.1% to 0.2% (at 5% significance level). The alpha of inverse volatility weighted portfolio is the highest among three smart-beta portfolios.

Overall, we see that smart-beta portfolios achieve higher returns than the market capitalization weighted portfolio. But they remain vulnerable to broad market downturns, as they still have large drawdowns (36%-58%). Let us now investigate whether market timing reduces these drawdowns and whether there is a cost in terms of expected returns to be paid.

4.2. Impact of market timing on portfolio performance

Is it possible to increase the performance both in terms of returns, risk and drawdown by switching the portfolio allocation between equities and a cash investment? This is the next question that we study. To answer this question, firstly we will present performances of portfolios using Faber's method and then consider six market timing investment strategies based on the expected return (and risk) of three estimated regime-switching (RS) models with two types of return target. The first RS model based investment assumes static transition probabilities and is denoted by RS_static. The second one (RS_dynamic) innovates by considering time-varying transition probabilities based on a composite index of state variables. RS_static and RS_dynamic invest in equities when the market is flat or rising. The third model is based on the same model as RS_dynamic but adds an additional risk control layer in the investment and invests in equities only if the market is flat or rising and the expected shortfall calculated at 95% is below 10% (RS_EScontrol). For each of these three regime switching strategies, two types of minimum return target are studied: a fixed return target of -2% and a trend following return target $\tilde{\kappa}_{t-1}$ in (1.2). The main results are reported in Table 2, where we report the different return and risk performance measures, as well as descriptive statistics on the frequency and persistence of the switches between equities and cash. More precisely, it reports i) number of switches between the equity investment and full cash position, and ii) the percentage of months the strategy invests in cash (1 month LIBOR based USD).

4.2.1. Market timing based on Faber's method using rolling price averages

The performances of portfolios using Faber's method are presented in column 3 of Table 2. Regarding to the market capitalization weighted portfolio, the portfolio using Faber's method has a higher return (11.13% p.a vs 10.32%), significantly lower volatility (10.41% vs 14.53%), significantly higher Sharpe ratio (1.07 vs 0.71) and lower drawdowns (maximum drawdowns is

17% vs 50% of the benchmark). This result is in line with the research of Faber (2007). In terms of three other smart-beta portfolios (inverse volatility, equal and fundamental value weighted portfolio), using Faber's method, portfolios have the same or lower returns (11.19%-11.72% vs 11.45%-12.94%), significantly lower volatilities (8.53%-11.47% vs 10.92%-16.51%), higher Sharpe ratios (0.98-1.34 vs 0.76-1.05) and much lower drawndowns (maximum drawdowns between 16%-21% vs 36%-55%) than the buy and hold benchmarks. Overall, using Faber's method, portfolios have much lower volatilities, drawdowns along with the same or little lower returns and the risk adjusted returns are all higher than benchmarks. In the next section, we will see whether the market timing strategies using the regime switching models improve portfolio performances.

4.2.2. Market timing based on three different regime switching models

In this section, we study the impact of the design of the regime switching model based investments on the portfolio performance.

Model 1: The constant (static) transition probabilities regime switching model.

The results for the regime switching model with static transition probabilities and a static -2% target in terms of minimum required return are reported in column 4 in Table 2. Note that the RS portfolios using static transition probabilities has similar performance to the benchmarks (the equity-only investment in column 2 in Table 2) in terms of a similar return, volatility and Sharpe ratios. The similar performance is due to the low number of switches between equity and cash (from 0 to 12 times) and low percentage of time investing in cash (less than 4%). For the inverse volatility weighted portfolio, there is even no switch at all using the static transition probabilities regime switching model. In comparison with the Faber's method, portfolios using this market timing strategy underperform (returns are lower, volatilities are significantly higher and Sharpe ratios are also lower). Overall, the market timing strategy using static transition probabilities and a fixed -2% threshold only slowly reacts to the market changes.

A solution is to applying the trend following return target $\tilde{\kappa}_{t-1}$ as can be seen in column 5 in Table 2. Portfolios using static transition probabilities regime switching model then have better performances than using the fixed return target in terms of risk adjusted return and drawdowns. The impact on return is mixed (higher for the market capitalization portfolio and fundamental value weighted portfolio but lower for others) but the volatility is significantly lower and the Sharpe ratios are substantially higher. Also the drawdowns are improved compared to the buy-and-hold in the equity benchmarks. The risk adjusted returns (Sharpe ratio) are nevertheless lower than those using Faber's method.

The results of regime switching model with static transition probabilities are therefore not satisfactory yet. It motivates us to study the usage of dynamic transition probabilities in regime switching model to do market timing.

Model 2: Time-varying (dynamic) transition probabilities regime switching model

As described in Subsection 3.2, we consider seventeen candidate state variables as drivers for the transition probabilities. Before analyzing market timing strategy with dynamic transition probabilities regime switching model, we investigate which variables are selected as drivers for time-varying transition probabilities and when.

Variables selected as drivers for the transition probabilities. It is natural to expect that the variable selection will depend on the market regime. In Table 4, we show the obtained variable selection in terms of number of months for three bullish and two bearish periods in our sample, Jan 1991 to Jul 2014, when variables are selected (in terms of BIC) in the multivariate linear regression model.

Table 4: Variable selection in the multivariate regression model that predicts the market capitalization weighted portfolio returns based on lagged state variables[3]. For each variable, the number of months the variable is selected is reported

Group of variables	Period	Jan-1991-Aug-2000	Sep-2000-Sep-2002	Oct-2002-Oct-2007	Nov-2007-Feb-2009	Mar-2009-Jul-2014	Full sample
	Duration (in months)	**116**	**25**	**61**	**16**	**65**	**283**
Macro economic variables	Real GDP growth	4	19	13	12	16	**64**
	Inflation	15	3	1	9	24	**52**
	PMI	3	0	15	2	4	**24**
Macro-financial variables	TED spread	18	9	6	0	1	**34**
	Short term yield	4	3	15	0	23	**45**
	Long term yield	14	0	6	0	5	**25**
	Term spread	11	7	10	4	11	**43**
	Credit spread	32	0	2	5	37	**76**
	VIX	21	8	35	0	0	**64**
	Change of VIX	0	0	1	0	56	**57**
	Variance risk premium	0	0	0	0	49	**49**
	CAY	28	23	58	12	34	**155**
	CAPE	54	23	57	16	52	**202**
	Fear index	9	0	26	9	63	**107**
Smart-beta momentum variables	One month return	17	9	1	0	1	**28**
	Av. of three month return	1	0	0	1	0	**2**
	Price spread	54	16	3	2	45	**120**

Note: The order of variables matches with it in Table 1; the first, the third and the fifth periods are bullish while the second and the fourth periods are bearish; variables are selected in terms of BIC.

We find that during the bullish periods, macro-financial variables and smart-beta momentum variables are mostly selected while the group of macro-economic variables appears in the selection during the bearish periods (Sep-2000 to Sep-2002 and Nov-2007 to Feb-2009), particularly the real GDP growth variable in this group is usually selected. Two variables (CAPE and CAY) in the group of macro-financial variables are actively selected in all five periods. Overall, macro-financial variables (consumption wealth ratio (CAY), cyclically adjusted price-earnings (CAPE) and fear index) and market variables (price spread) play the most important roles in predicting stock return.

Performance gains of market timing strategy using time-varying (dynamic) transition probabilities. Let us now return to the key question of this part: is the strategy of dynamic transition probabilities regime switching better than the buy and hold strategy and Faber's strategy in terms of improving the portfolio performance? We find that, compared to the benchmark (column 2 in Table 2), the portfolios that invest based on the expected return under the RS model with dynamic transition probabilities regime switching model and the fixed return target (column 6 in Table 2) performs better than the benchmarks with insignificantly higher returns (0.5%-3% p.a) and significant lower volatility (0.6% to 3% at 1% and 5% significant level), their Sharpe ratios are insignificantly higher than the benchmarks. Their downside risks are also lower than the benchmarks (maximum drawdowns are in range of 29%-38% vs 36%-58%) (except for the inverse volatility weighted portfolio, their maximum drawdown is 36%, the same as the benchmark). Compared to the market timing strategy of Faber, portfolios using this strategy have higher return but also significantly higher volatilities, so their Sharpe ratios are indeed lower than those using Faber's strategy.

[3] We only present the table of variable selection for the market capitalization weighted portfolio. Except for the inverse volatility weighted portfolio, those of other equity portfolios are similar and available upon request.

Using the trend following return target, portfolio performances are presented in column 7 in Table 2. Portfolios yield higher returns, significantly lower volatilities, significantly higher Sharpe ratios, and drawdowns also lower than the benchmarks. Importantly, their returns are consistently higher than the benchmarks and volatilities, drawdowns are always lower than the benchmark. In comparison with Faber's strategy, these portfolios always have higher returns (11.67%-13.42% vs 11.13%-11.72%), higher Sharpe ratios (1.11-1.39 vs 0.98-1.34), comparable volatility (8.42%-12.08% vs 8.53%-11.47%) and little lower drawdowns (maximum drawdown of 15%-19.88% vs 16.22%-21.18%). It confirms that the usage of dynamic transition probabilities regime switching and the trend following return target improve portfolio performances compared to the buy-hold and Faber's strategy. Next, we will investigate the benefit of risk control in market timing using dynamic transition probabilities regime switching model.

Model 3: Time-varying (dynamic) transition probabilities regime switching with risk control model.

We have seen the benefit of using dynamic transition probabilities in market timing strategy in the previous paragraphs. In this part, we will investigate the benefits of including also a limit on downside risk in the investment by answering the question: does the market timing strategy using the dynamic transition probabilities perform better than buy-and-hold strategy and Faber's strategy? Firstly, we will check this strategy with the usage of fixed return target. Compared to the benchmarks, the portfolios using dynamic transition probabilities regime switching with the control of expected shortfall (column 8 in Table 2) have mixed results of returns and significant lower volatility than the benchmarks. Their downside risks are also lower than the benchmarks. Meanwhile, this strategy does not perform better than Faber's strategy in terms of lower returns (except for the inverse volatility weighted portfolios), lower Sharpe ratios, higher volatilities and drawdowns.

The performance improves further when, instead of the investing based on comparing the predicted mean $\mu_{t|t-1}$ with the fixed level of -2%, we compare the conditional mean $\mu_{t|t-1}$ with the trend following return target threshold $\tilde{\kappa}_{t-1}$. The resulting performance is shown in column 9 of Table 2. Using this strategy, portfolios usually have higher returns (except for the equally weighted portfolio), higher Sharpe ratios and lower volatility and drawdowns than the benchmarks. Their performances are slightly better than those using Faber's strategy in terms of higher returns (except for the market capitalization weighted portfolio), higher Sharpe ratio (1.01-1.47 vs 0.98-1.34) and lower volatility (8.24%-11.67% vs 8.53%-11.47%), drawdowns (15%-19.88% vs 16.22%-21.18%).

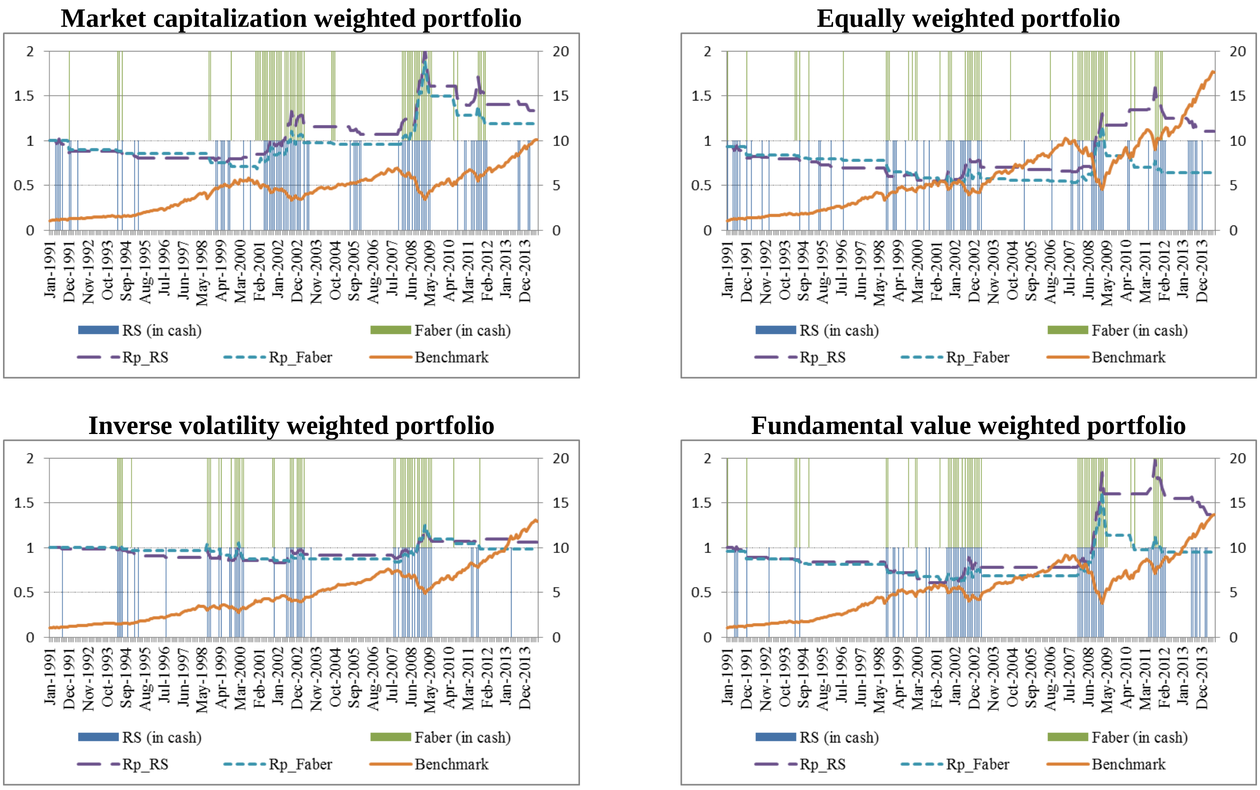
Compared to portfolios using strategy without risk control (column 7 in Table 2), portfolios with risk control have lower volatility. These results are expected as this market timing strategy focuses on controlling potential risk. In exchange, the portfolio returns are lower than portfolios without risk control (except for the inverse volatility weighted portfolio where the return is higher: 11.72% vs 12.08%). It can be explained by the fact that percentages of period investing in cash of the strategy with a risk control are higher than regime switching strategy without a risk control.

In summary, we see that the usage of the trend following return target is better than the fixed one. Secondly, among three regime switching model, market timing strategy using dynamic transition probabilities perform best among three market timing strategies using regime switching model: static transition probabilities, dynamic transition probabilities and dynamic transition probabilities with a risk control. In the next part, we will compare it to the simple tactical asset allocation strategy of Faber using rolling price averages in detail.

4.2.3. Regime switching model based strategy versus Faber's method using rolling price averages

The regime switching models are complex to implement, compared to the tactical allocation strategy based on rolling price averages. Portfolios using Faber's strategy invest in the smart beta portfolios as long as their cumulative return is greater than their 10 month simple moving average cumulative return, otherwise in cash. The disadvantage of Faber's strategy is however that it is mechanical and assumes a certain deterministic behavior of financial markets, which is intuitively difficult to swallow given the time-varying stochastic features of the market. We now investigate whether the complexity pays off in terms of better performance.

Figure 3: The cumulative return of the benchmarks and relative performance of the switching portfolios based on rolling prices (Faber) and the regime switching model with dynamic transition probabilities and the trend following return target



Note: Each figure shows, on the right-hand side axis, the full line is the cumulative return of the equity-only portfolios weighted portfolio (the benchmark). On the left hand side axis, the long-dash line and short-dash line are the relative performance of the dynamic regime switching portfolio with the trend following return target (Rp_RS) and Faber's strategy (Rp_Faber) versus the benchmark, respectively. The cash investments of portfolios using dynamic transition probabilities regime switching model and trend following return target are indicated with the lower vertical bars (RS (in cash)) while the upper vertical bars indicate the cash investments of portfolios using Faber's strategy (Faber (in cash)).

Compared to the market timing strategy using regime switching model, portfolios applying Faber's strategy have always a lower annualized return (11.13% vs 11.67%, 11.39% vs 11.72%, 11.19% vs 13.42% and 11.72% vs 13.23% for the market cap weighted, inverse volatility weighted, equally weighted and fundamental value weighted portfolios, respectively). As shown in Figure 3 their relative performance lines are always below those of RS strategies using dynamic transition probabilities regime switching model. In terms of risk, portfolios using Faber's strategy have slightly lower volatility but a slightly higher downside risk than portfolios using dynamic transition probabilities regime switching. Overall, portfolios using regime switching model with dynamic transition probabilities and the trend following return targets have higher risk-adjusted returns (Sharpe ratios) than those using Faber's strategy (1.12 vs 1.07, 1.39 vs 1.34, 1.11 vs 0.98 and 1.26 vs 1.12). It is further interesting to note that portfolios using dynamic transition probabilities regime switching model usually switch between equity and cash more frequently than portfolios applying Faber's strategy. Also note in Figure 3 that they make different timing decisions, explaining the difference in return performance. Overall, we thus find that, for our sample of S&P 500 stocks over the period 1991-2014, the complexity of the regime switching model based tactical allocations seems to pay off in terms of higher annualized return, Sharpe ratio and lower drawdowns with respect to the more simple trend following rules based on moving average price of Faber's strategy.

5. Conclusion

Smart beta portfolios are increasingly popular. By an alternative weighting and selection with respect to the market capitalization weighted portfolio, they typically achieve a superior diversification, but remain vulnerable to broad market downturns. We examine tactical rules to switch timely between equity and cash investments based on an underlying regime-switching model with macro-economic, macro-financial and momentum variables as drivers for time-varying transition probabilities. The investment universe analyzed consists of the S&P 500 stocks over the period 1991-2014. We answer two questions. First, how does an alternative smart beta weighting scheme affect the portfolio performance compared with the market capitalization weighted portfolio. Second, we investigate whether the smart beta portfolio performance can be improved by switching the portfolio allocation between equities and a cash investment.

We find that, compared to the traditional market capitalization weighted portfolio, equally weighted, inverse volatility weighted and fundamental value weighted portfolio yields a higher compound return over the period. Except for the inverse volatility weighted portfolio, this comes at a higher investment risk. The higher return and lower risk of the inverse volatility weighted portfolio is consistent with the ample literature documenting the low risk anomaly for the universe we analyze. We further show that the tactical investment decisions based on the regime switching model with time-varying transition probabilities and the trend following minimum expected return target improves the portfolio performance. It leads to investment decisions that are more reactive to changes in the market conditions and a substantially better risk-adjusted performance and lower drawdowns.

Acknowledgements

Financial support from the Doctiris program of the Brussels Institute for Research and Innovation (Innoviris) is gratefully acknowledged. The authors also would like to thank Arkadi Avanesyan and Marjan Wauters for their support.

References

Ang, A. and Timmermann, A. (2012). Regime changes and financial markets. *Annual Review of Financial Economics*, 4(1): 313–337.

Arnott, R., Hsu, J., and Moore, P. (2005). Fundamental indexation. *Financial Analysts Journal*, 61(2): 83–89.

Baker, M., Bradley, B., and Wurgler, J. (2011). Benchmarks as Limits to Arbitrage: Understanding the Low Volatility Anomaly. *Financial Analysts Journal*, 67(1), 40-54.

Baker, N. L. and Haugen, R. A. (2012). Low risk stocks outperform within all observable markets of the world. Available at SSRN 2055431.

Barber, B. M. and Lyon, J. D. (1997). Firm size, book-to-market ratio, and security returns: A Holdout Sample of Financial Firms. *The Journal of Finance*, 52(2): 875-883.

Bauer, R., Koedijk, K., and Otten, R. (2005). International evidence on ethical mutual fund performance and investment style. Journal of Banking and Finance, 29(7), 1751-1767.

Blanchard, O. and Watson, M. (1982). Bubbles, rational expectations and financial markets. *P. Wachtel, ed., Crises in the economic and financial structure, Lexington, MA: Lexington Books*, 295-315.

Bollerslev, T., Tauchen, G., and Zhou, H. (2009). Expected stock returns and variance risk premia. *Review of Financial Studies*, 22(11): 4463-4492.

Boudt, K., Danielsson, J., Koopman, S. J., and Lucas, A. (2012). Regime switches in the volatility and correlation of financial institutions. *National Bank of Belgium Working Paper*, (227).

Broda, S. A. and Paolella, M. S. (2011). Expected shortfall for distributions in finance. *Statistical Tools for Finance and Insurance* (pp. 57-99). Springer Berlin Heidelberg.

Danielsson, J. (2011). Risk and crises. *VoxEU.org 18 February.*

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of Finance*, 52(1): 57-82.

Cecchetti, S. G., Lam, P. S., and Mark, N. C. (1993). The equity premium and the risk-free rate: Matching the moments. *Journal of Monetary Economics*, 31(1): 21-45.

De Boer, S. and Norman, J. H. (2014). Tactical timing of low volatility equity strategies. Available at SSRN 2492270.

DeMiguel, V., Garlappi, L., and Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy. *The Review of Financial Studies*, 22(5): 1915–1953.

Diebold, F.X., Lee, J.-H., and Weinbach, G. (1994), "Regime switching with time-varying transition probabilities," in C. Hargreaves (ed.), Nonstationary Time Series Analysis and Cointegration. (Advanced Texts in Econometrics, C.W.J. Granger and G. Mizon, eds.), 283-302. Oxford: Oxford University Press.

Diebold, F. and Mariano, R. (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics*, 13(3): 253–263.

Dutt, T. and Humphery-Jenner, M. (2013). Stock return volatility, operating performance and stock returns: International evidence on drivers of the "low volatility" anomaly. *Journal of Banking and Finance*, 37(3): 999–1017.

Engle, R. F. and Colacito, R. (2005). Testing and valuing dynamic correlations for asset allocation. *Journal of Business and Economic Statistics*, 24(2), 238–253.

Estrella, A. and Mishkin, F. S. (1998). Predicting US recessions: financial variables as leading indicators. *Review of Economics and Statistics*, 80(1): 45-61.

Faber, M. T. (2007). A quantitative approach to tactical asset allocation. *The Journal of Wealth Management*, 9(4): 69-79.

Fama, E. F. and Schwert, G. W. (1977). Asset returns and inflation. *Journal of Financial Economics*, 5(2): 115-146.

Fama, E. F. (1990). Term-structure forecasts of interest rates, inflation and real returns. *Journal of Monetary Economics*, 25(1), 59-76.

Fama, E. F. and French, K. R. (1992). The cross- section of expected stock returns. *The Journal of Finance*, 47(2): 427-465.
Gilchrist, S., Yankov, V. and Zakrajšek, E. (2009). Credit market shocks and economic fluctuations: Evidence from corporate bond and stock markets. *Journal of Monetary Economics*, 56(4): 471-493.
Giot, P. and Petitjean, M. (2011). On the statistical and economic performance of stock return predictive regression models: an international perspective. *Quantitative Finance*, 11(2): 175-193.
Guidolin, M. and Timmermann, A., (2008). International asset allocation under regime switching, skew, and kurtosis preferences. *Review of Financial Studies*, 21 (2): 889–935.
Guo, H. and Whitelaw, R. F. (2006). Uncovering the risk–return relation in the stock market. *The Journal of Finance*, 61(3): 1433-1463.
Hamilton, J.D. (2008). Regime-switching models, In: Durlauf, S.N., Blume, L.E. (Eds.), The New Palgrave Dictionary of Economics, Second Edition. Palgrave Macmillan, 1–15.
Hamilton, J. D. (1989). A New Approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57(2): 357-84.
Jobson, J. and Korkie, B. (1981). Performance hypothesis testing with the Sharpe and Treynor measures. *Journal of Finance*, 36(4): 889–908.
Konishi, S. and Kitagawa, G. (2008). Bayesian information criteria. In: Information Criteria and Statistical Modeling. Springer Series in Statistics. Springer New York, 211–237
Kothari, S., Leone, A. and Wasley, C. E., 2005. Performance matched accrual measures. *Journal of Accounting and Economics*, 39(1): 163–197.
Kritzman, M., Page, S. and Turkington, D., 2012. Regime shifts: Implications for dynamic strategies. *Financial Analysts Journal*, 68(3): 22–39.
Ledoit, O. and Wolf, M. (2008). Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance*, 15(5): 850–859.
Leote De Carvalho, R., Lu, X. and Moulin, P., 2012. Demystifiying equity risk-based strategies: A simple alpha plus beta description. *Journal of Portfolio Management*, 38(3): 56–70.
Lettau, M. and Ludvigson, S. (2001). Consumption, aggregate wealth, and expected stock returns. *Journal of Finance*, 56(3): 815-849.
Maheu, J. M. and McCurdy, T. H. (2000). Identifying bull and bear markets in stock returns. *Journal of Business and Economic Statistics*, 18(1): 100-112.
Marron, J. S. and Wand, M. P. (1992). Exact mean integrated squared error. *The Annals of Statistics*, 20(2): 712-736.
Memmel, C. (2003). Performance hypothesis testing with the Sharpe ratio. *Finance Letters*, 1(1): 21-23.
Meucci, A. (2013). Estimation and stress-testing via time-and market-conditional flexible probabilities. Available at SSRN 2312126.
Norden, L. and Weber, M. (2009). The co-movement of credit default swap, bond and stock markets: An empirical analysis. *European Financial Management*, 15(3): 529-562.
Perold, A. F. (2007). Fundamentally flawed indexing. *Financial Analysts Journal*, 63(6): 31–37.
Price, K.V., Storn, R.M., and Lampinen J.A. (2006). Differential Evolution - A practical approach to global optimization. *Berlin Heidelberg: Springer-Verlag*. ISBN 3540209506.
Schaller, H. and Norden, S. V. (1997). Regime switching in stock market returns. *Applied Financial Economics*, 7(2): 177-191.
Shaikh, I. and Padhi, P. (2014). The implied volatility index: Is 'investor fear gauge'or 'forward-looking'?. *Borsa Istanbul Review*, 15(1): 44-52.
S&P Dow jones index. S&P Low volatility index methodology (2014). Retrieved from: http://us.spindices.com/indices/strategy/sp-500-low-volatility-index .
Taboga, M. (2011). Under/Over valuation of the stock market and cyclically adjusted earnings. *International Finance*, 14(1): 135-164.

Van Dijk, D. and Franses, P. H. (1999). Modeling multiple regimes in the business cycle. *Macroeconomic dynamics*, 3(3): 311-340.

Walkshausl, C. and Lobe, S. (2010). Fundamental indexing around the world. *Review of Financial Economics*, 19(3): 117–127.

Wolfers, J. and Zitzewitz, E. (2004). Prediction markets (No. w10504). *National Bureau of Economic Research*.

Yamai, Y. and Yoshiba, T. (2005). Value-at-risk versus expected shortfall: A practical perspective. *Journal of Banking and Finance*, 29(4): 997-1015.

Appendix

In the Appendix we provide more details on the inference regarding the probabilities to be in each regime using Hamilton's filter and the calculation of the expected shortfall.

Hamilton's filter. Hamilton's filter aims at estimating from the observed returns y_t the probability to be in each regime, given all information available in the return data up to time t:

$$\xi_{j,t|t} = \Pr(S_t = j|\Omega_t;\ \theta), \qquad \textbf{(1. 15)}$$

where $\Omega_t = \{y_t,\ y_{t-1}, \dots, y_1, y_0\}$ denotes the set of observations obtained as of date *t*; and θ: is a vector of population parameters. The key magnitudes to perform this iteration are the densities under the regimes:

$$\eta_{j,t} = f(y_t|S_t = j, \Omega_{t-1};\ \theta) = \frac{1}{\sqrt{2.\pi}\,\sigma_{S_t}} \exp\left(-\frac{\left(y_t - \mu_{S_t}\right)^2}{\sigma_{S_t}^2}\right). \qquad \textbf{(1. 16)}$$

Given the input from (1.15), we can iteratively compute

$$\xi_{j,t|t} = \frac{\sum_{i=1}^{2} p_{ij,t}\xi_{i,t-1|t-1}\eta_{j,t}}{f(y_t|\Omega_{t-1};\ \theta)}, \qquad \textbf{(1. 17)}$$

where $p_{ij,t}$ is the transition probability from regime *i* to regime *j* on date *t* and $f(y_t|\Omega_{t-1};\ \theta)$ is the conditional density of the *t*-th observation:

$$f(y_t|\Omega_{t-1};\ \theta) = \sum_{i=1}^{2}\sum_{j=1}^{2} p_{ij,t}\xi_{i,t-1|t-1}\eta_{j,t}. \qquad \textbf{(1. 18)}$$

Calculation of expected shortfall of stock returns under the RS model. As we assume the returns follow a normal distribution in the two regimes, the conditional return distribution is a mixture of normals, and, as explained in Broda and Paolella (2011), expected shortfall can be computed in two steps. First it requires to compute the corresponding quantile based on numeric techniques. Then an explicit expression for the expected shortfall is calculated. More precisely, let μ_j, σ_j be the mean and sigma of the return series y_t under regime *j*; $y_t \sim N(\mu_j, \sigma_j^2)$; *(j = 1, 2)*. The cdf corresponding to (1.18) is:

$$F(y_t|\Omega_{t-1};\ \theta) = \sum_{j=1}^{2} \xi_{j,t|t-1}.\,\Phi\left(y_t;\ \mu_j, \sigma_j^2\right), \qquad \textbf{(1. 19)}$$

where, $\xi_{j,t|t-1}$ stands for predicted probability in regime *j* on date *t* (cfr (1.12)). It follows that the ɣ- quantile of y_t, $q_{y_t,ɣ}$, can be determined by solving the equation ɣ- $F(q_{y_t,ɣ};\ \theta) = 0$ (throughout the paper ɣ is set to 0.05, corresponding to a 95% ES calculation). Letting $c_{j,t} = \left(q_{y_t,ɣ} - \mu_j\right)/\sigma_j$, $\Phi\left(c_{j,t}\right)$ and $\varphi\left(c_{j,t}\right)$ are the standard normal distribution and density function evaluated at $c_{j,t}$, respectively. Then, the expected shortfall of the return is given by:

$$ES_{ɣ}\left(y_t;\ \mu_j, \sigma_j, \xi_{j,t|t-1}\right) = \sum_{j=1}^{2} \frac{\xi_{j,t|t-1}\,\Phi\left(c_{j,t}\right)}{ɣ}\left\{\mu_j - \sigma_j\,\frac{\varphi\left(c_{j,t}\right)}{\Phi\left(c_{j,t}\right)}\right\}. \qquad \textbf{(1. 20)}$$